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கல்வி அமைச்சு
Ministry of Education



# G.C.E.(A.L) Support Seminar - 2023

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

**10 E I**

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மூன்று மணித்தியாலம்
Three hours

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மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

**Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.**

| Index Number | | | | | | | |
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## Instructions:

- *This question paper consists of two parts;*
  **Part A** (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
- **Part A:**
  *Answer* ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:**
  *Answer* ***five*** *questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

**For Examiners' Use only**

| (10) Combined Mathematics I | | |
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| **Part** | **Question No.** | **Marks** |
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[see page two

## Part A

**1.** Using the **principal of Mathematical induction** prove that $\sum_{r=1}^{n} 2^r = 2(2^n - 1)$ for all $n \in \mathbb{Z}^+$

**2** Sketch the graph of $= \left||x-2|-2\right|$ . Hence or otherwise solve the equation $\left||x-2|-2\right| = \frac{x}{2}$

**3** Shade the region $R$ that represents the complex number $Z$ satisfying the condition $0 \leq ArgZ \leq \frac{\pi}{3}$ in an Argand Diagram.

Also find the least Value of $|iZ + 2|$ in the region $R$.

**4** **Write down** the binomial expansion of $(1 + x)^n$ in ascending powers of $x$. Given that the coefficient of $x^2$ in the expansion $(1 + x + ax^2)^7$ is 14. Show that $a = -1$.



[see page four

**5** Show that $\lim_{x \to \frac{\pi}{4}} \frac{2\sqrt{x}-\sqrt{\pi}}{\sin(x-\frac{\pi}{4})} = \frac{2}{\sqrt{\pi}}$

**6** If $f(x) = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x}$ , then find $\frac{d[f(x)]}{dx}$. Hence, deduce $\int \tan^{-1}\sqrt{x}\, dx$. The region enclosed by the curves $y = \sqrt{\tan^{-1}\sqrt{x}}$ , $x = 3$ and $y = 0$ is rotated about the $x-$ axis through $2\pi$ radians. Show that the volume of the solid thus generated is $\frac{\pi}{3}\left(4\pi - 3\sqrt{3}\right)$ cubic units.



[see page five

**7** A curve C is given by the parametric equations $x = a\cos\theta$ and $y = b\sin\theta$ for $(0 \leq \theta \leq \pi)$. Show that the equation of the normal to the curve C, at point P, is $ax\sec\alpha - by\operatorname{cosec}\alpha + b^2 - a^2 = 0$.

Also find the normal to the curve C, at point $\left(-\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ on the curve C.

......................................................................................................................................................

**8** The straight line $l \equiv y - mx = 0$ passes through the point of intersection of two straight lines $4x + 3y - k = 0$, where $k$ is constant and $5x - 12y + 7 = 0$. Find the value of $m$ in terms of $k$. Further, given that the line, $l = 0$ is perpendicular to the line $x + y = 0$. Find the values of $m$ and $k$.

......................................................................................................................................................



[see page six

**9** A circle $S$ with centre on the y-axis intersects the circle $x^2 + y^2 = 9$ orthogonally and the circle $x^2 + y^2 + x - 7y + 5 = 0$ bisects the circumference of the circle $S$. Show that there are two such circle $S$, and find their equations.

**10** Given that $tanA = \frac{5}{12}$ and $sinB = \frac{4}{5}$; Where A and B are such that $\pi < A < \frac{3\pi}{2}$ and $\frac{\pi}{2} < B < \pi$. Find the value of $sin(A + B)$



[see page seven

# G.C.E.(A.L) Support Seminar - 2023

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 S I

## Part B

✱ Answer **five** questions only.

**11 (a)** Write down the sum and the product of the roots of quadratic equation $ax^2 + bx + c = 0$, in terms of $a, b$ and $c$ where $a, b, c \in \mathbb{R}, a \neq 0$

Given that $f(x) \equiv x^2 - p^2qx + q^2$ where $p, q \in \mathbb{R}^+$ and roots of the equation $f(x) = 0$ are α and β.

(i) Find $\alpha^{\frac{3}{2}} + \beta^{\frac{3}{2}}$ in terms of $p$ and $q$.

(ii) If α and β are real, then find the least integer value of $p$.

(iii) For the above $p$ value find the quadratic equation in terms of $q$, whose roots are $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$.

**(b)** Let $P(x) \equiv 2x^3 + x^2 - 2x + \lambda$ ; where $\lambda \in \mathbb{R}^+$

(i) If $\lambda$ is zero of the polynomial $P(x)$, find $\lambda$

(ii) If $-\lambda$ is zero of the polynomial $P(x)$, find $\lambda$

(iii) For the value of $\lambda$ which satisfies both (i) and (ii), write down the polynomial $P(x)$ and express $P(x)$ as a multiple of linear factors.

(iv) Find the remainder, when $P(x) + 3x + 2$ is divided by $x^2 + 1$.

**12 (a)** An institution has 8 cars and there are parking facilities in two rows, 4 cars in each row in the park.

(i) Find the number of ways in which 8 cars can be parked.

(ii) Find the number of ways in which 8 cars can be parked, if the first place in the first row is to be reserved for chairman's car and a place in the first row for the car of secretaries.

(iii) If the first place in the first row should be given to either one of the two cars of chairman's or secretaries, and if the other car should also have a place in the first row, find the number of ways in which the cars can be parked.



[see page eight

**(b)** Find the value of the constants A and B such that,

$$\frac{r^2+3r-1}{(r^2-r+1)(r^2+r+1)} = \frac{Ar+B}{r^2-r+1} - \frac{Ar+2B}{r^2+r+1} \quad ; \text{ where } r\epsilon\mathbb{Z}^+.$$

If, $U_r = \dfrac{r^2+3r-1}{(r^2-r+1)(r^2+r+1)}$ then determine $f_r$ such that $U_r = f_r - f_{r+1}$

Hence, show that $\displaystyle\sum_{r=1}^{n} U_r = 2 - \frac{(n+2)}{n^2+n+1}$

Is this series convergent? Justify your answer.

If, $\displaystyle\sum_{r=1}^{n} U_r < 2 - \frac{11}{91}$ then find greatest value of $n$.

**13 (a)** If $A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, show that any real matrix B which commutates with A, under multiplication, can be written in the form $\lambda A + \mu I$, where $\lambda$ *and* $\mu$ are real numbers and $I$ is the identity matrix of order 2. Find the value of $\lambda$ *and* $\mu$ *when* $B = A^2$ Hence Find $A^{-1}$.

**(b)** By Factorizing $Z^6$-1, completely solve the equation $Z^6 = 1$.

If $Z_1$ and $Z_2$ are any two distinct roots of the equation $Z^6 = 1$, show by reference to an Argent diagram, or otherwise, that the three possible values of $|Z_1 - Z_2|$ are 1, 2 and $\sqrt{3}$.

**(c)** By using De moivre's theorem for positive integer $n$,

Show that $\left(\dfrac{1+sin\theta+icos\theta}{1+sin\theta-icos\theta}\right)^n = \cos\, n\left(\dfrac{\pi}{2}-\theta\right) + isin\, n\left(\dfrac{\pi}{2}-\theta\right)$

Deduce that, $\left(\dfrac{1+i}{1-i}\right)^{2n} = (-1)^n$

**14 (a)** Let $f(x) = \dfrac{x(x+3)}{(x+1)^2}$ for $x \neq -1$

Show that $f'(x)$ the first derivative of $f(x)$ with relative to $x$, is given by

$f'(x) = -\dfrac{(x-3)}{(x+1)^3}$

.

Hence, find the intervals on which $f(x)$ is decreasing and the intervals on which $f(x)$ is increasing.

Obtain the coordinates of the turning point of $f(x)$.

It is given that $f''(x) = \dfrac{2(x-5)}{(x+1)^4}$ for $x \neq -1$.

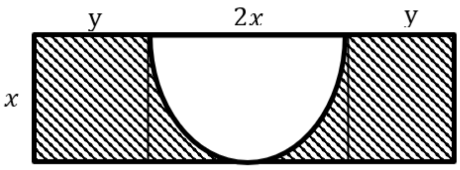
Find the coordinates of the point of inflection on the graph of $y = f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, turning point and point of inflection.



[see page nine

**(b)** The shaded region shown in the figure is obtained by removing a semicircular lamina of radius $x\,m$ from a rectangle of length $2(x+y)\,m$ and width $x\,m$.




The area of the rectangle is $8\pi\,m^2$. Show that the perimeter p of the shaded region, measured in meters, is given by $P = \pi\left(x + \frac{16}{x}\right)$

**15** **(a)** Determine the values of constants A, B and C such that

$x^4 + 1 = A(x^4 - 1) + B(x^2+1)(x+1) + C(x^2+1)(x-1) - (x^2-1)$ for $x \in \mathbb{R}$ ,

hence, find $\int \frac{x^4+1}{x^4-1}\,dx$

**(b)** **(i)** If $y = x + \cos x \sin^3 x$ show that $\frac{dy}{dx} = 1 + 3\sin^2 x - 4\sin^4 x$.

Given that $I = \int_0^{\frac{\pi}{2}} (x + 3x\sin^2 x - 4x\sin^4 x)\,dx$. By using above result and using integration by parts, Show that $I = \frac{1}{8}(\pi^2 - 2)$

**(ii)** Further given that,

$J_1 = \int_0^{\frac{\pi}{2}} (1 + 3\cos^2 x - 4\cos^4 x)\,dx$

$J_2 = \int_0^{\frac{\pi}{2}} (x + 3x\cos^2 x - 4x\cos^4 x)\,dx$

Using the result $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Show that $I = \frac{\pi}{2}J_1 - J_2$

Now given that $\frac{d}{dx}(x - \sin x\cos^3 x) = 1 + 2\cos^2 x - 4\cos^4 x$

show that $J_2 = \frac{1}{8}(\pi^2 + 2)$, deduce the value of $J_1$.

**(c)** Using the substitution $\sqrt{x^3+1} = t$ , Evaluate $\int_0^2 \frac{x^8}{\sqrt{x^3+1}}\,dx$.



[see page ten

**16** $l_1: x - \sqrt{3}y + 1 + k = 0$ and $l_2: x + \sqrt{3}y + 1 - k = 0$ are two given straight lines passing through the point $(-1, 3)$ show that $k = 3\sqrt{3}$.

For that value of k, find the equations of the angle bisectors between the straight lines $l_1 = 0$ and $l_2 = 0$.

Let, $l$ be the acute angle bisector of $l_1$ and $l_2$. Show that the point $A \equiv (2,3)$ lies on the line $l = 0$.

Find the equation of the circle $S$ with centre $A$ and the length of the diameter is 3 units.

Find the perpendicular distance from the point $A$ to the line $l_1 = 0$, hence find the equation of the tangent drawn from $(-1, 3)$ to the circle $S$.

From a point $P$ on the line $l = 0$, two tangents are drawn to the circle $S$ so that they are perpendicular to each other.

Show that there are two such points for $P$ and in each case find the coordinates.

Further, find the area of the quadrilateral which enclosed by the tangents.

**17 (a) (i)** Write down $\cos(A + B)$ In terms of $\cos A, \cos B, \sin A, \sin B$ and obtain an expression for $\cos 3A$ in terms of $\cos A$.

**(ii)** Determine constants $\lambda$ and $k$ such that,

$$\frac{2\cos 3x - 4cos^5x + 3cos^3x}{cosx(1 + sin^2x)} = \lambda \cos 2x + k$$

Hence, find the maximum and minimum values of

$$f(x) = \frac{2\cos 3x - 4cos^5x + 3cos^3x}{cosx(1 + sin^2x)}$$

and sketch the graph of $y = f(x)$ for $x \epsilon [-\pi, \pi]$

**(b)** A point $P$ is inside the triangle ABC, such that $P\hat{A}B = P\hat{B}C = P\hat{C}A = \alpha$

By applying **Sine Rule** for suitable triangles, write down two expressions for $PC_1$ and show that $cot\alpha = cotA + cotB + cotC$

**(c)** Solve the equation $2tan^{-1}(cosx) = tan^{-1}(2cosecx)$ for $x \epsilon \left(0, \frac{\pi}{2}\right)$.

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கல்வி அமைச்சு
Ministry of Education

# G.C.E.(A.L) Support Seminar - 2023

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

පැය තුනයි
மூன்று மணித்தியாலம்
*Three hours*

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

**Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.**

| Index Number | | | | | | | |
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## Instructions:

* *This question paper consists of two parts;*
  **Part A** (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

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* In this question paper, $g$ denotes the acceleration due to gravity.

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**For Examiners' Use only**

| (10) Combined Mathematics II | | |
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| **Part** | **Question No.** | **Marks** |
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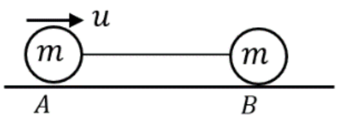
[See page two]

## Part A

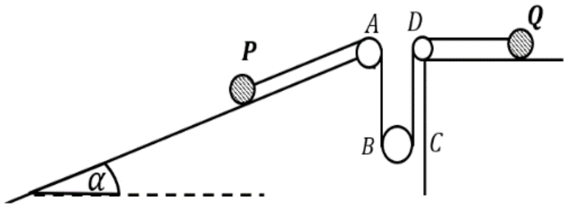
**1.** Two particles A and B each of mass m are connected by a light inextensible string. The particle A is given a velocity u along the direction of $\overrightarrow{AB}$. The coefficient of restitution between A and B is $e$. Find the velocities of A and B after the collision. Also find the impulsive tension in the string when it becomes taut.




**2.** Two equal particles P and Q are projected at the same instant respectively from points A and B at the same level on the ground. where $AB = a$. The particle P from A towards B with velocity $u$ at $45^0$ above AB and the particle Q from B towards a with velocity $v$ at $60^0$ with respect to BA. If the particles just pass through a point, then show that $u : v = \sqrt{3} : \sqrt{2}$ and find $u$ in terms of $a$ and $g$.

**3.** In the diagram is a light inextensible string attached to a particle of mass $m$ placed on a fixed smooth plane inclined at to the horizontal. The string passes over a fixed small smooth pulley.

In the diagram PABCDQ is a light in extensible string attached to a particle of mass m placed on a fixed smooth plane inclined at $\alpha$ to the horizontal. The string passes over a fixed small smooth pulley at A and under a smooth pulley of mass $\boldsymbol{M}$ and passes over a fixed small smooth pulley at B and attached to a particle Q of mass $\boldsymbol{m}$ placed on fixed horizontal plane. PA is along a line of greatest slope AB and DC are vertical and DQ is horizontal. The system is released from rest with the string taut. Write down equations sufficient to determine tension in the string and accelerations of the particle and the movable pulley.

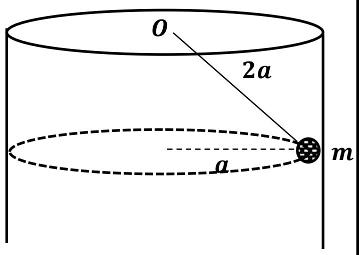



**4.** A car of mass M kg is travelling along level road at constant velocity $u\,m\,s^{-1}$ with the engine working at maximum power of 3H W. Find the constant resistance force to motion of the car now the car. Now the car moves down a straight rode slope of 1 in 30 and the engine works at the same rate. If the resistance to motion is unchanged, show that maximum speed attains the car is $\frac{90\,Hu}{90H-Mgu}$ $m\,s^{-1}$ $\left(H > \frac{mgu}{90}\right)$.



[See page four]

**5.** A smooth hollow cylinder of internal radius $a$ is fixed its axis vertical. One end of a light inextensible string of length $2a$ is fixed to a centre of a point O on the lid of the cylinder. The other end of the string is attached to a particle of mass $m$ which moves in a horizontal code on the surface of the cylinder with constant angular speed $\omega$ . (see in the figure)




Show that the tension in the string is $\frac{2mg}{\sqrt{3}}$ and find the magnitude of the force exerted on the particle by the cylinder.

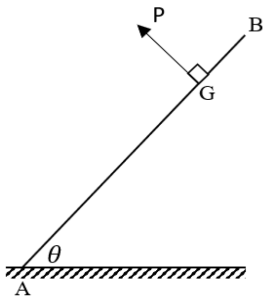
**6.** In the usual notation ska $\underline{i} + \underline{j}$, $4\underline{i} + \underline{j}$ and $6\underline{i} + 3\underline{j}$ are the position vectors of points $A, B$ and $C$ respectively, with respect to a point O. Also $D$ be a point on AB such that $AD: DB = 1: 2$. Find the position vector of the point $D$ with respect to the point O. Show that the points $O, D$ and $C$ are collinear.



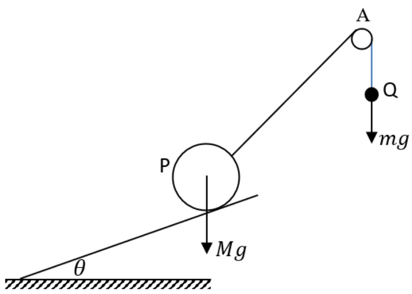
[See page five]

**7.** The figure shows a non-uniform rod AB of weight W, whose lower end A rests on rough horizontal ground. The rod is inclined at $\theta$ to the horizontal and rests in equilibrium attached by an inextensible string perpendicular to the rod AB at its centre of granting G, where AG: GB = 3: 1.

Indicate clearly, what are the forces act on the rod AB. Given that reaction A is S and the string exerts a force P, then find the values of P and S in terms of W and $\theta$




**8.** In the diagram a light inextensible string attached to a uniform spere P of mass M placed on a fixed rough plank inclined at an angle $\theta$ to the horizontal. The string passes over a fixed small pulley at A and other end attached to a particle A of mass m. AQ is vertical and other part of the string is inclined at $\theta$ to the vertical.






[See page six]

**9.** Events $A, B, C$ occur so that $P(A) = 8k$, $P(B) = 2k$ and $P(C) = k$; $k \neq 0$. Given also that $(A/B) = P(B/C) = 0$ , $P(A/C) = P(A)$ and $(A \cup B \cup C) = \frac{11}{12}$,

Find, (i) $P(A \cap B), P(B \cap C)$ and $P(A \cap B \cap C)$

(ii) $P(C \cap A)$ in forms of $k$ and the value of $k$

**10.** Each of the ten observations $1004, 1008, 1000, 1008, 996, 992, 1000, 1008, 1008, 1000$ are expressed in the form $1000 - 4x$. Find the mean and standard deviation of the values of $x$ and deduce the mean of the ten observations given.



[See page seven]

# G.C.E.(A.L) Support Seminar - 2023

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

## Part B

※ Answer five questions only.

(In this question paper, $g$ denotes the acceleration due to gravity.)

**11.** (*a*) A ball P attached to a balloon starts rising vertically upwards from rest with uniform acceleration $\lambda g$ from point O on the ground. When the balloon has reached a height $h$ with the velocity v, the ball freely detached and it moves under gravity. When the balloon has reached further height $h$, another ball Q is projected vertically upwards from the same point O with the velocity $v$. It is given that the velocity of the ball is $v$ when it is detached, while $\lambda$ is a positive real constant.

(i) Draw the velocity- time graphs for the motion of each ball on the same diagram.

(ii) Show that $v = \sqrt{2\lambda gh}$ when it detached from the balloon.

(iii) Show that the time taken for the two balls to meet from the instant Q is projected is $\left(h + \frac{v^2}{2g}\right)$

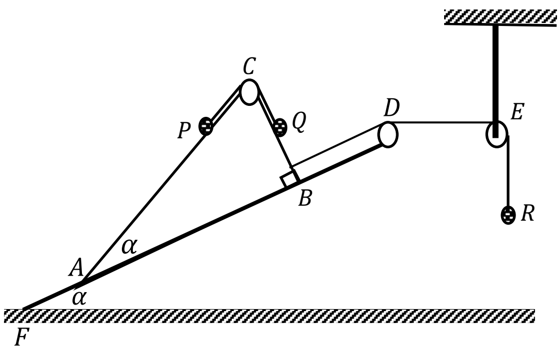
(*b*) A ship A sailing with uniform velocity V in the direction of the South observes a boat P sailing the direction α degrees East of South. At the same instant, the ship A also observes another boat Q sailing the direction $\alpha$ degrees East of North.

(i) Draw in the same diagram, the velocity triangle for A and P and the velocity triangle for A and Q.

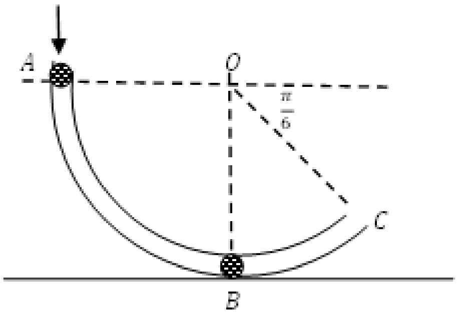
(ii) Find the speed of P and Q relative to A.

(iii) Given that initially the ship A is due East at a distance 24km from P and due North at am distance 6 km from Q, Find the distance between A and Q when the distance A and P is Least.

**12.** (*a*) The triangles ABC in the figure is vertical cross sections through the center of gravity of a smooth uniform wedge of mass $3m$ with $A\hat{B}C = \frac{\pi}{2}$ $and$ $B\hat{A}C = \alpha$. Such that the face AB placed on a line of greatest slope of a smooth fixed inclined plane. The inclined plane makes an angle $\alpha$ to the horizontal. The two ends of a light inextensible string passing over a small smooth small pulley fixed at C are attached to the particles P and Q of masses 2m and m respectively. One end of another light inextensible string is attached, point of the wedge and passing over smooth small pulleys fixed at D and E. The other end is attached to the particle R of mass m. The string DE is in the horizontal position. At the initial position, the particles P and Q are held at AC and CB respectively such that, $AF = a$ and the strings are taut. Here F is the point where the inclined plane meets the horizontal plane. The system is release from rest, assume that the wedge reaches the F before the particles P and Q reach foot of the wedge. Obtain the equations sufficient to determine the time taken by the wedge to reach the point F in terms of a and g.



***(b)*** A thin smooth tube ABC is the shape of a circular arc of the radius $a$ and centre O is fixed in a vertical plane with OA horizontal and the lowest point B of the tube touching a fixed horizontal floor as shown in the figure. The radius OC makes an angle $\frac{\pi}{6}$ with the downward horizontal.




Show that the radius to the composite body makes an angle $0 \leq \alpha \leq \frac{\pi}{3}$ with OB, the speed $v$ of the composite body is given by $v^2 = 2ga\left(\frac{1}{(1+\lambda)^2} + \cos\theta - 1\right)$.

If $\lambda \leq \sqrt{2} - 1$, show that the composite body never leaves the tube and it oscillates along an arc of the tube.

Given that $\lambda = \sqrt{2} - 1$ , Show that the reaction on the composite body from the tube is $\sqrt{\frac{3}{2}}$mg when the body becomes instantaneously rest.

**13.** A thin light elastic spring, of natural length$3l$, stands vertically with its lower end O fixed and carries a particle P of mass $m$ fastened to its upper end. This particle is resting in equilibrium at the point A at a length $4l$ vertically above O, by a constant fixed force $3mg$ acts vertically upwards on the particle . Show that the modulus of elasticity of the spring is $6mg$.

Now the particle is gently released from the point A show that the equation of the motion of the particle is $\ddot{x} + \frac{2g}{l}(x - \frac{5l}{2}) = 0$: where $x$ is the displacement of the particle from O at time $t$ for $3l < x < 4l$. Rewrite the equation in the form $\ddot{X} + \omega^2 X = 0$ where X = $x - \frac{5l}{2}$ and $\omega^2 = \frac{2g}{l}$.

Assuming that $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude C of this simple harmonic motion. When it is at point B at height $3l$, vertically above point O. At the instant when the particle P is at B, another particle Q of mass $m$ is gently collides and coalesces with P. Show that the composite body, will begin to move vertically downward with speed $\sqrt{gl}$ .

Let, point D be the lowest point reached by the composite body, show that the motion of equation is $\ddot{y} + \frac{3g}{l}\left(y - \frac{8l}{3}\right) = 0$, where $y$ is the verticle displacement of the particle from O, $2l < y < 3l$.

Assuming that the solution of the above equation is of the form $y = \frac{8l}{3} + \alpha\cos\omega t + \beta\sin\omega t$, find the values of constants $\alpha, \beta$ and $\omega$.

Hence, find the centre and amplitude of the simple harmonic motion performed by the composite body from B to D.

**14.** **(*a*)** The position vector of point A with respect to an origin O is $\sqrt{3}\underline{i} + \underline{j}$, Where $\underline{i}$ and $\underline{j}$ have the usual meaning. Let B the point such that OB=10 units and $B\hat{O}A = 60^0$.

Take $\overrightarrow{OB} = \alpha\underline{i} + \beta\underline{j}$ ; $\alpha \neq 0$. Find $\overrightarrow{OB}$

(i) Let C be the point on OB such that $\overrightarrow{AC} = \frac{\sqrt{3}}{2}\underline{i} - \frac{5}{2}\underline{j}$ . Find OC: CB

Give that D is the midpoint of OA, find $\overrightarrow{BD}$.

(ii) Let, E be the point such that $\overrightarrow{AE} = \frac{10}{17}\overrightarrow{AC}$ find $\overrightarrow{BE}$. Show that B, E and D point are collinear.

**(*b*)** ABCD is a trapezium in which the side AB is parallel to DC, AB is perpendicular to BD, $D\hat{A}B = 60^0$ and $C\hat{A}B = 30^0$ .

Forces of magnitudes $\mu p, 2p, 3\sqrt{3}p$ and $\lambda p$ act along $AB, AD, CA$ and DB respectively, in the directions indicated by the order of the letters.

(i) Show that the system is not in equilibrium any values of $\lambda$ and $\mu$

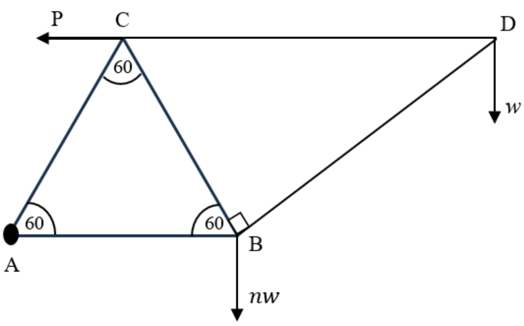
(ii) If the resultant force of the system is along AD, then find $\lambda$ and $\mu$

(iii) Now, a force $\alpha p$ and couple $G$ acting in the same plane are added to the system in the direction through $\overrightarrow{CD}$. If the resultant force of the system is along DB, find the values of α and G.

**15. (*a*)** The weights of the uniform rods AB, BC, CD and DA of equal lengths are 2W, 2W, 3W and W respectively. A quadrilateral frame is formed by joining rods freely at A, B, C and D. This is kept in the shape of a square by a light inextensible string connecting the centre of the gravity of the rods AB, BC and suspended freely from the joint A.

Show that the tension in the string is 9W. Also find, magnitude of reactions at the joints B and C for the rod BC.

**(b)** The framework shown in figure consists of five light rods smoothly joined their ends A, B, C and D. The rods AB, BC and AC are equal lengths, weighed $nw$ and $w$ are hung from B and D respectively. The framework smoothly hinged at A and kept in position of equilibrium by a horizontal force $P$ applied at C with the rods ab and CD are in horizontal.




(i) Show that $P = \left(\frac{2n+5}{\sqrt{3}}\right)w$.

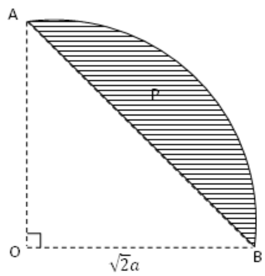
(ii) Draw a stress diagram. Using Bows notation and determine the stresses in the rods classifying them on tensions or thrust.

(iii) Given that the maximum stress force in the rod BC is $10\sqrt{3}\,w$, then show that $n \leq 14$.

**16. (*a*)** (i) Using integration show that the center of mass of a uniform circular sector of a circle of radius $r$, subtending at angle $2\alpha$ at the centre is at a distance $\frac{2r\sin \propto}{3 \propto}$ from the centre.

(ii) Show that centre of mass of a uniform hollow cone of base radius $a$ and height h is at a distance $\frac{2}{3}h$ from its vertex.

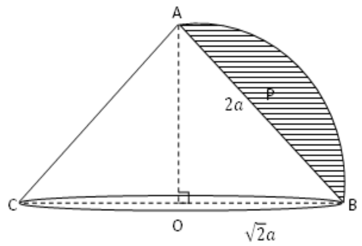
**(*b*)** The figure shows the uniform lamina P obtained by removing an isosceles triangular lamina from a uniform circular sector of a circle of radius $\sqrt{2}\,a$ subtending at angle $\frac{\pi}{2}$ at its centre O. Show that the centre of mass is at a distance $\frac{4a}{3(\pi-2)}$ from O on its symmetric axis.






[See page ten]

**(c)** A composite body is formed by a hollow thin right circular cone of base diameter $\sqrt{2}a$ , slant height $2a$ and the lamina P fixed along the slant height of the cone is shown in the figure. Given that mass of the cone is five forms of mass of the lamina. Then find the position of centre of mass of the composite body, taking OB and OA as X and Y axes respectively. If the composite body is suspended freely from the point A, show that the angle $tan^{-1}\left[\frac{5\pi-8}{2(9\pi-19)}\right]$ made by OA with the downward vertical.




**17. (a)** In analyzing of the records of a children's hospital revealed a few probability measures for the following events for male child chosen at random from th0se who are receiving treatment.

Event *A*: The child has Asthma.

Event *B*: The child has high blood pressure.

Event *C*: The child has diabetes.

It is given that the events *A*, *B*, *C* are mutually Independent and that,

$P(B) = 0.3, P(A \cup B) = 0.37$ and $P(C) = 0.2$

(i) Show that $P(A) = 0.1$

(ii) Find $P(B'/A')$, where $A'$ and $B'$ denote the complements of *A* and *B* *respectively*.

(iii) Find the probability that the child has diabetes but has neither high blood pressure nor asthma.

(iv) Given that the child has just one of these ailments, find the probability that is asthma

**(b)** The following table shows the distances to the nearest Kilometers travelled to work 120 employees of a company.

| Distance | Number of Employees |
|---|---|
| 0 -10 | 10 |
| 10-20 | 19 |
| 20-30 | 43 |
| 30-40 | 25 |
| 40-50 | 8 |
| 50-60 | 6 |
| 60-70 | 5 |
| 70-80 | 3 |
| 80-90 | 1 |

(i) Using the transformation $y_i = \frac{1}{10}(x_i - 45)$ estimated the mean and standard deviation of the distribution.

(ii) The company decided to transfer the employees who travel more than 50 km, to closer branches of the company which are closer to them. Estimate the interquartile travel range of the distances travel to work by the remaining employees after the transfers are made.



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