නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

NEW

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

ගණිතය I
கணிதம் I
Mathematics I

07 E I

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number | | | | | | | |

Instructions:

* *This question paper consists of two parts;* **Part A** (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
* **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $A = \{x \in \mathbb{R} : |x+1| \leq 2\}$ and $B = \{x \in \mathbb{R} : |x-1| > 1\}$. Find $A \cap B$, $A \cup B$ and $A \cap B'$.

2. Let A and B be subsets of a universal set S. Show that $(A \cup B) \cap (A \cap B)' = (A \setminus B) \cup (B \setminus A)$.

[see page three

0232

3. Show that the compound proposition $(p \wedge q) \vee r$ and the compound proposition $(p \Rightarrow \sim q) \Rightarrow r$ are logically equivalent.

4. Using the method of contradiction, prove that if $n^2 + 6n + 3$ is even, then n is odd.

[see page four

5. Solve the equation $\log_3 x = 2 - \log_3(6 - x)$ for x.

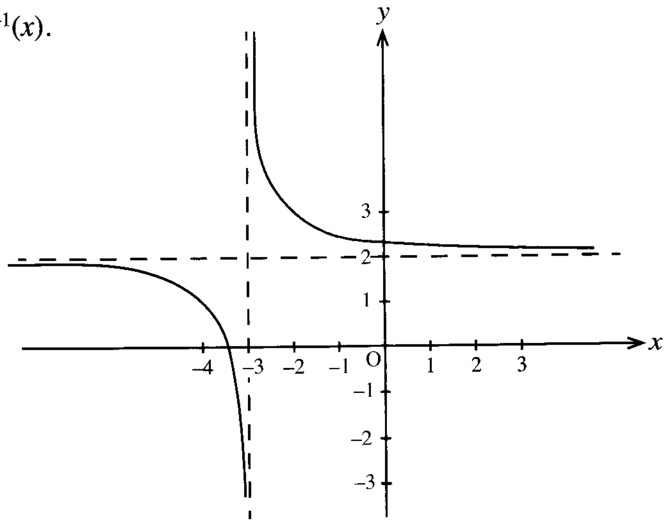
When x takes this value, solve the equation $x^y = 2 - x^{-y}$ for y. (Hint: Use the substitution $u = x^y$.)

6. Find all real values of x satisfying the inequality $x + \frac{6}{x+1} > 4$.

[see page five

7. The graph of $f(x)=\dfrac{1}{x+a}+b$ is shown in the diagram. Using the information given there, write down the values of the constants a and b, and find $f^{-1}(x)$.

Given that $g(x) = x - 5$, solve $f^{-1}(g(x)) = 4$.



....

8. Write down the equation of the straight line l passing through the point $A \equiv (0, 3)$ with gradient -2. The line l meets the line $y = mx$ at the point B, where $m\ (\neq -2)$ is a constant. Find the x–coordinate of B in terms of m.

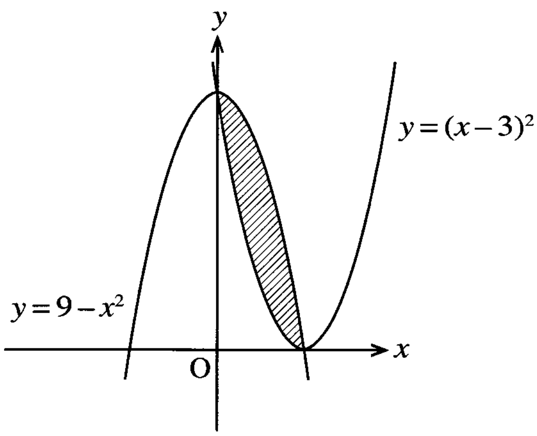
Given that the area of the triangle OAB is $\frac{9}{2}$ square units, where O is the origin, find the possible values of m.

....

[see page six

9. The radius r m of a right circular cylinder is increasing at a rate of 0.5 m s^{-1} and its height h m is decreasing at a rate of 0.2 m s^{-1}. Show that the rate of change of the volume V m^3 of the cylinder is given by $\frac{dV}{dt} = \frac{\pi r}{5}(5h - r)$.

10. Find the area of the region enclosed by the curves $y = 9 - x^2$ and $y = (x-3)^2$.



නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

ගණිතය I
கணிதம் I
Mathematics I

07 E I

Part B

✻ *Answer **five** questions only.*

11. (*a*) A survey was carried out using 100 students in a class to find out which branches of mathematics they liked from amongst Algebra and Geometry. It was found that the number of students who liked Geometry was 10 more than twice the number of students who liked Algebra. It was also found that 80 students liked only one branch and 10 students did not like both.

Find the number of students who liked

(i) Algebra

(ii) Geometry

(iii) both Algebra and Geometry.

(*b*) Using truth tables, determine whether each of the following compound propositions is a tautology or a contradiction.

(i) $(p \wedge q) \wedge (q \Rightarrow \sim p)$

(ii) $(p \wedge q \wedge r) \vee (p \wedge q \wedge (\sim r)) \vee (\sim (p \wedge q))$

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n} r(3r+2) = \frac{n}{2}(n+1)(2n+3) \text{ for all } n \in \mathbb{Z}^+.$$

(*b*) Let $U_r = \dfrac{r^2+r-1}{(r+1)^2(r+2)^2}$ for $r \in \mathbb{Z}^+$.

Verify that $U_r = \dfrac{r}{(r+1)^2} - \dfrac{(r+1)}{(r+2)^2}$ for $r \in \mathbb{Z}^+$.

Show that $\displaystyle\sum_{r=1}^{n} U_r = \frac{1}{4} - \frac{(n+1)}{(n+2)^2}$ for $n \in \mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Deduce that $\displaystyle\sum_{r=20}^{\infty} U_r = \frac{20}{441}$.

13. (*a*) Let $k\ (\neq 0)$ be a real constant. It is given that the quadratic equation $2kx^2 + 12x + 2k - 5 = 0$ has real roots. Show that $2k^2 - 5k - 18 \leq 0$.

Find the maximum and the minimum of possible values of k.

Let α and β be the roots of the equation $2kx^2 + 12x + 2k - 5 = 0$.

Find the quadratic equation whose roots are $2(\alpha + \beta)$ and $3\alpha\beta$.

(*b*) Let $f(x) = x^3 + px^2 + q$ and $g(x) = x^3 + qx^2 - p$, where p and q are real numbers. It is given that $(x+2)$ is a factor of $f(x)$ and that when $g(x)$ is divided by $(x+1)$, the remainder is -8.
Find the values of p and q.

For these values of p and q, find the least value of $f(x) - g(x)$.

14. (*a*) Let $a, b \in \mathbb{R}$. The expansion of $(1+ax)^8$, in ascending powers of x, discarding the terms involving powers of x greater than two is $1 + 24x + bx^2$. Show that $a = 3$ and $b = 252$.

Hence, find an approximate value for $(1.03)^8 + (0.97)^8$.

(*b*) A person wants to take a loan of Rs. 2000000 from a bank, to be paid back in 10 years. The bank charges an annual interest of 6% compounded monthly. Let Rs. A_n be the outstanding amount after paying the n^{th} installment at the end of the n^{th} month, where $n \leq 120$.
Show that $A_1 = 1.005A - x$, where A is the loan amount and x is the monthly installment.
Obtain expressions for A_2 and A_3, and write down A_n in terms of A, x and n.
Hence, find the value of x.

15. Let $A \equiv (1,\ 1)$ and $B \equiv (5,\ 9)$.

Find the equation of the straight line AB and show that the point $C \equiv (4,\ 2)$ does not lie on the line AB.
The line perpendicular to AB and passing through C, intersects AB at the point D.
Find the coordinates of D and show that $AD{:}DB = 1{:}3$.

Also, find the coordinates of the point E such that $ADCE$ is a rectangle.

Let F be the point of intersection of the line AB and the line $x + y = k$. The line passing through the point F and parallel to the line AC passes through the point E. Find the value of the constant k.

16. (*a*) Evaluate $\displaystyle\lim_{x \to 2} \frac{x^4 - 16}{\sqrt{x} - \sqrt{2}}$.

(*b*) Differentiate each of the following with respect to x:

(i) $(2 + 3x)^5\,(1 + x^2)^{10}$ (ii) $\dfrac{\ln x}{3\ln x + 1}$ (iii) $\sqrt{x}\, e^{-(x^2-1)}$

(*c*) A closed rectangular box needs to be constructed such that the length of the base is 3 times its width. It costs 100 rupees per square meter for the top and the bottom faces, and 60 rupees per square meter for the sides of the box. If the volume of the box must be 60 m^3, show that the cost C (in rupees) to make the box is given by $C = 600x^2 + \dfrac{9600}{x}$, where x m is the width of the base of the box.

Determine the value of x that minimizes the cost to make the box.

17. (*a*) Using the method of **integration by parts**, find $\int x^3(\ln x)^2\,dx$.

(*b*) The following table gives the values of the function $f(x) = \ln(1 + x^2)$, correct to three decimal places, for values of x between 1 and 2.5 at intervals of length 0.25.

x	1.00	1.25	1.50	1.75	2.00	2.25	2.50
$f(x)$	0.693	0.941	1.179	1.402	1.609	1.802	1.981

Using **Simpson's rule**, find an approximate value for $I = \int_1^{2.5} \ln(1+x^2)\,dx$.

Hence, find an approximate value for $\int_1^{2.5} \ln\left(e^{2x}\sqrt{1+x^2}\right)dx$.

* * *

නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

NEW

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

ගණිතය II
கணிதம் II
Mathematics II

07 E II

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

- ✻ *This question paper consists of two parts;*
  Part A (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
  Part A:
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
  Part B:
  *Answer **five** questions only. Write your answers on the sheets provided.*
- ✻ *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- ✻ *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
- ✻ *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $a, b, c \in \mathbb{R}$.

Show that $\begin{vmatrix} a & a & 2a+b+c \\ b & a+2b+c & b \\ a+b+2c & c & c \end{vmatrix} = -2(a+b+c)^3$.

2. Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 2 \\ -1 & 2 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 1 & 3 \end{pmatrix}$. Find **AB** and **BC**.

Verify that $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$.

0232

3. The mean and the standard deviation of a set of 10 observations are 5 and 10, respectively. Find the sum and the sum of squares of these observations.
If another observation of value 5 is added to this set, find the new values of the mean and the standard deviation.

4. The mean, median and the standard deviation of a distribution are 28, 32 and 5, respectively. Calculate Karl Pearson's coefficient of skewness and describe the shape of the distribution.
Is the mean a fair measurement of the central tendency for this distribution? Give reasons for your answer.

[see page four

5. The speed of vehicles travelling on a certain section of a highway, is normally distributed with mean 90 $km\,h^{-1}$ and standard deviation 10 $km\,h^{-1}$. Find the probability that the speed of a randomly selected vehicle is between 85 $km\,h^{-1}$ and 100 $km\,h^{-1}$.

6. It is found from previous records that 10% of the bolts produced by a machine are defective. If 5 bolts produced by this machine are chosen at random, find the probability that

(i) exactly 3 bolts are defective,

(ii) more than 2 bolts are **non-defective**.

7. In a group consisting of 30 cricketers, 20 have played for club A and 15 have played for club B. Every cricketer has played for at least one of these clubs. Find the probability that a cricketer selected at random has played for club B, given that he has played for club A.

8. Let A and B be two events of a sample space S such that $P(A)=\frac{3}{8}$, $P(A\cap B)=\frac{1}{8}$ and $P(A\cup B)=\frac{3}{4}$. Find

(i) $P(B)$, (ii) $P(A'\cap B)$ and (iii) $P(A'|B)$.

[see page six

9. The probability mass function of a discrete random variable X is given below:

x	1	2	3	4	5
$P(X=x)$	p	$2p$	p	$2p$	p

Find the value of the constant p and show that $E(X) = 3$.

Let Y be the random variable given by $3X-4$. Find $P(Y>X)$.

....

....

....

....

....

....

....

....

....

....

....

....

....

10. A continuous random variable X has probability density function $f(x)$ given by

$$f(x)=\begin{cases} kx-x^2 & , \text{ if } 0\le x\le 1, \\ 0 & , \text{ otherwise}, \end{cases}$$

where k is a constant.

Show that $k=\frac{8}{3}$ and find $E(X)$.

....

....

....

....

....

....

....

....

....

....

....

....

....

නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

NEW

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2020
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2020
General Certificate of Education (Adv. Level) Examination, 2020

ගණිතය II
கணிதம் II
Mathematics II

07 E II

Part B

* *Answer five questions only.*

11. A factory manufactures tables and chairs. The production of each item requires three operations: cutting, assembling and finishing.

For cutting, assembling and finishing, the maximum number of hours that can be used are 600, 160 and 280, respectively. The following table gives the number of hours required for each operation in producing each item and the profit per item sold.

	Number of hours for cutting	Number of hours for assembling	Number of hours for finishing	Profit (in thousands of rupees)
Table	5	1	1	12
Chair	6	2	4	15

The factory wishes to maximize the profit.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the solution of the problem formulated in part (i) above.

(iv) Due to shortage of storage space, the factory has to limit the total number of tables and chairs produced to at most 108. Find the decrease in the profit due to above limitation, if the factory still wishes to maximize the profit.

12. (*a*) Let $\mathbf{A} = \begin{pmatrix} 4 & 7 \\ -1 & -2 \end{pmatrix}$. Write down $\mathbf{A}^{-1}$.

Let $\mathbf{B} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$.

Find the matrix **C** such that **AC** = **B** and show that

$$\mathbf{AC} - \mathbf{CA} = \begin{pmatrix} 20 & 43 \\ -11 & -20 \end{pmatrix}.$$

Find the matrix **D** such that **AC** − **DA** = **O**, where **O** is the zero matrix of order 2.

[See page eight

(*b*) Let $a \in \mathbb{R}$. Write the pair of **simultaneous** equations

$$(a-5)x + 3y = a$$
$$-4x + (a+2)y = 1$$

in the form **PX** = **Q**, where $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$, and **P** and **Q** are matrices to be determined.

Express $\Delta = \begin{vmatrix} (a-5) & 3 \\ -4 & (a+2) \end{vmatrix}$ as a quadratic function of a.

Show that the roots of the equation $\Delta = 0$ are $a = 1$ and $a = 2$.

Show that the above pair of equations has

(i) infinitely many solutions when $a = 1$,

(ii) no solution when $a = 2$,

(iii) a unique solution when $a = 3$.

13. (*a*) An unbiased cubic die with faces marked 1, 2, 2, 3, 3, 4 is tossed twice. Let A be the event that the sum of the numbers obtained is 4 and B be the event that the sum of the numbers obtained is even.

Find $P(A)$, $P(B)$ and $P(A \mid B)$.

(*b*) Four digits from the set of digits {1, 2, 3, 4, 5, 6} are chosen without replacement and a 4-digit number is made.

(i) How many different 4-digit numbers can be made?

(ii) How many of these 4-digit numbers start with 3 or 5?

(*c*) A team of four people must be selected from a group of four males and two females.

(i) How many different teams of four people can be selected?

(ii) Find the probability that both females are selected to these teams.

14. A box X contains 4 red cards and 6 blue cards. A box Y contains 3 red cards and 2 blue cards. A biased coin with $\frac{2}{3}$ as the probability of getting a head is tossed. If the outcome is a head, 2 cards are drawn from the box X, at random without replacement, and if it is a tail, 2 cards are drawn from the box Y, at random without replacement. Find the probability that

(i) both cards drawn are red,

(ii) at least one of the cards drawn is red,

(iii) the two cards drawn are of different colours,

(iv) the two cards drawn are of different colours, given that at least one of the cards drawn is red.

[See page nine

15.(*a*) The time X, measured in minutes, between consecutive arrivals of buses to a certain bus stop is exponentially distributed with probability density function

$$f(x)=\begin{cases} \lambda e^{-\lambda x} & , \quad x>0, \\ 0 & , \quad \text{otherwise}, \end{cases}$$

where $\lambda\,(>0)$ is a parameter.

If the mean number of buses that arrive at the bus stop in an hour is 12, find the value of λ.

(i) After a bus arrives at the bus stop, find the probability that the time taken for the next bus to arrive at the bus stop is

(α) between one minute to three minutes,

(β) less than five minutes.

(ii) If it is given that five minutes has already passed from the arrival of a bus to the bus stop, find the probability that it takes at least an additional two minutes for the next bus to arrive.

(*b*) A continuous random variable X is uniformly distributed over the interval $[a, b]$.

Find the values of a and b such that $P(X < 16) = 0.4$ and $P(X > 21) = 0.2$.

16. Hundred students faced an entrance test. The frequency distribution of the marks they obtained is given in the following table:

Marks	frequency
0 – 20	15
20 – 40	20
40 – 60	40
60 – 80	15
80 – 100	10

(i) Estimate each of the following:

(a) the mean,

(b) the standard deviation,

(c) the median,

(d) the inter quartile range and

(e) the mode

of the marks.

(ii) After rescrutiny, it was discovered that the marks of two answer scripts should be changed as follows:

Marks before rescrutiny	Marks after rescrutiny
50	62
70	75

Find the mean of the new distribution of marks.

17. The duration of activities of a project and the flow of activities are given in the following table:

Activity	**Preceding Activity (Activities)**	**Duration (in Weeks)**
A	–	03
B	A	08
C	A	05
D	A	03
E	B	06
F	C	03
G	E, F	04
H	D, F	06
I	G, H	03

(i) Construct the project network.

(ii) Prepare an activity schedule that includes earliest start time, earliest finish time, latest start time, latest finish time and float for each activity.

(iii) Find the total duration of the project.

(iv) What are the activities that can be delayed without extending the total duration of the project?

(v) Write down the critical path of this project.

(vi) Suppose that the duration of the activity D has to be extended by two weeks due to an unexpected matter. Determine whether the project could still be completed within the total duration calculated in part (iii) above.

* * *