නව නිර්දේශය/புதிய பாடத்திட்டம்/New Syllabus

NEW

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය I கணிதம் I **Mathematics I**	**07**	**E**	**I**

28.08.2019 / 0830 – 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

Instructions:

* *This question paper consists of two parts;* **Part A** (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
* **Part A:** *Answer* ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:** *Answer* ***five*** *questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
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	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $A = \{x \in \mathbb{R}: |x-2| \geq 2\}$ and $B = \{x \in \mathbb{R}: |x-1| < 3\}$ be subsets of $\mathbb{R}$. Find $A \cap B$ and $A \cup B'$.

2. Let A and B be subsets of a universal set S. The set $A \setminus B$ is defined, in the usual notation, by $A \setminus B = A \cap B'$. Show that $A \setminus B = B' \setminus A'$ and $(A \setminus B) \setminus C = A \setminus (B \cup C)$.

[see page three

3. Show that the compound propositions $(p \Rightarrow q) \vee (p \Rightarrow r)$ and $p \Rightarrow (q \vee r)$ are logically equivalent.

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4. Using the method of contrapositive, prove that if n^3+5 is odd, then n is even.

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5. Solve the simultaneous equations $2\log_9 x + \log_3 y = 3$ and $2^{x+3} - 8^{y+1} = 0$ for x and y.

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6. Find all real values of x satisfying the inequality $x \leq \frac{2}{x-1}$.

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7. Let $f(x)=x^3+1$ and $g(x)=ax+b$ for $x\in\mathbb{R}$, where a and b are real constants. It is given that $f(g(0)) = 2$ and $g(f(0)) = 3$. Find the values of a and b.

With these values for a and b, find $g^{-1}(x)$.

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8. Let $A\equiv(1, 2)$ and $B\equiv(9, 8)$. Find the equation of the perpendicular bisector l of AB.

Two points C and D are taken on l such that $ACBD$ is a square. Show that the area of the square $ACBD$ is 50 square units.

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9. The surface area of a closed rectangular box with a square base of side length x m and height h m is 100 m^2. If x is increasing at a rate of 6 m s^{-1} while keeping the surface area unchanged, find the rate at which h is changing when $x = 5$ m.

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10. Find the area of the region enclosed by the curve $y=(x-2)^2$ and the straight line $2x+y=7$.

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නව නිර්දේශය/புதிய பாடத்திட்டம்/New Syllabus

NEW

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය	I
கணிதம்	I
Mathematics	**I**

07 E I

Part B

* *Answer **five** questions only.*

11. (*a*) Eighty five students of a certain school have to face two pre-qualifying examinations to qualify for the final examination.

Number of students passed in the first pre-qualifying examination is equal to twice the number of students passed in the second examination. The number of students who passed exactly one examination is 70 and 5 students failed both examinations.

Determine the number of students who passed

(i) each pre-qualifying examination,

(ii) both examinations.

(*b*) Using truth tables, determine whether each of the following compound propositions is a tautology, a contradiction or neither.

(i) $[p \wedge (\sim q \Rightarrow \sim p)] \Rightarrow q$

(ii) $[p \wedge (p \Rightarrow q)] \wedge (\sim q)$

(iii) $\sim (p \wedge q) \Rightarrow (p \vee q)$

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n} \left(6r^2 - 2r - 1\right) = n\left(2n^2 + 2n - 1\right) \text{ for all } n \in \mathbb{Z}^+.$$

(*b*) Let $V_r = \dfrac{1}{(r+1)(r+2)}$ for $r \in \mathbb{Z}^+$.

Verify that $V_r = \dfrac{r+1}{r+2} - \dfrac{r}{r+1}$ for $r \in \mathbb{Z}^+$.

Show that $\displaystyle\sum_{r=1}^{n} V_r = \frac{n}{2(n+2)}$ for $n \in \mathbb{Z}^+$.

Also, find $\displaystyle\sum_{r=6}^{16} (2V_r + 3)$.

[*See page eight*

13. (*a*) Let $a \in \mathbb{R}$. Show that the roots of the equation $x^2 + ax - 1 = 0$ are real and distinct.

Let α and β be these roots. Find the quadratic equation that has $2\alpha + 1$ and $2\beta + 1$ as its roots.

(*b*) Let $f(x) = x^3 + 3x^2 + px + q$, where p and q are real numbers.

The remainder when $f(x)$ is divided by $(x - 1)$ is -12 and $(x - 2)$ is a factor of $f(x)$. Find the values of p and q.

Also, find the other linear factors of $f(x)$.

14. (*a*) Let $k \in \mathbb{R}$. The coefficients of x^{20} and x^{21} in the binomial expansion of $(1 + kx)^{23}$ are equal. Show that $k = 7$.

(*b*) By discarding the terms involving powers of x greater than 3, find an approximate value for $(1.7)^{23} + (0.3)^{23}$.

(*c*) A person opened a bank account by depositing Rs. 50 000 at the beginning of a month. He then deposited Rs. 20 000 at the beginning of every month for two years. The account pays 0.5% interest compounded monthly. Find the balance in the account after two years.

At the end of every month after this two year period, he withdrew Rs. 20 000 from the account. For how long will there be money left in the account for him to continuously withdraw Rs. 20 000 per month?

15. Show that there are two straight lines l_1 and l_2 passing through the point (–2, 8) and sum of the intercepts on the axes is 6.

A straight line meets the above two straight lines l_1 and l_2 at points P and Q respectively. If the midpoint of the line segment PQ is (1, 5), find the equation of the line PQ.

Show that the equation of the straight line passing through the point of intersection of straight lines l_1 and l_2, and perpendicular to PQ is $4y = x + 34$.

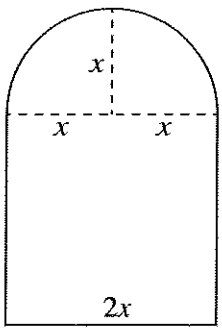
16. (*a*) Find $\lim_{x \to a} \frac{x^2 - a^2}{x^3 - a^3}$.

(*b*) Differentiate each of the following with respect to x:

(i) $\ln\left(x + e^{\sqrt{x}}\right)$ (ii) $\left(x + \sqrt{x^2 + a^2}\right)^3$ (iii) $\sqrt{\frac{1 + e^x}{1 - e^x}}$

(*c*) A window is in the shape of a rectangle surmounted by a semicircle as shown in the figure. The entire perimeter of the window is $(\pi + 4)$ m. By taking x m as the radius of the semicircle, show that the area of the window A m^2 is given by $A = k\left(2x - x^2\right)$, where $k = \frac{1}{2}(\pi + 4)$.

Find the value of x such that the area of the window is maximum.



|See page nine

17. (*a*) Using the method of **integration by parts**, evaluate $\int (x+1)^2 e^x \, dx$.

(*b*) The following table gives the values of the function $f(x)=\frac{1}{(2-x)^2}$ correct to four decimal places, for values of x between 0 and 1 at intervals of length 0.2.

x	0.00	0.20	0.40	0.60	0.80	1.00
$f(x)$	0.2500	0.3086	0.3906	0.5102	0.6944	1.0000

Using **Trapezoidal rule**, find an approximate value for $I = \int_0^1 \frac{1}{(2-x)^2} dx$, correct to three decimal places.

Find I using the substitution $u=2-x$ or otherwise and compare with the approximation obtained above.

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නව නිර්දේශය/புதிய பாடத்திட்டம்/New Syllabus

NEW

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය II கணிதம் II **Mathematics II**	**07**	**E**	**II**

29.08.2019 / 0830 – 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

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- ✻ **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
- ✻ *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- ✻ *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
- ✻ *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
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B	11	
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In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $\Delta = \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$, where a, b and c are distinct non-zero real constants. If $\Delta = 0$, show that $abc = -1$.

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2. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 2 \\ -1 & 4 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 0 \\ 3 & -1 \\ 2 & 1 \end{pmatrix}$. Find $A+B$, AC and BC. Verify that $(A+B)C = AC+BC$.

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3. The time X taken (in hours) for assembling a motorcycle follows a normal distribution with mean μ and standard deviation 5. If 10% of the motorcycles are assembled in less than 14 hours, find the mean μ.

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4. A company has two sections A and B with 50 and 60 employees in each, respectively. In a particular year the average and the standard deviation of the monthly wages in the two sections are given in the following table:

Section	Number of employees	Average monthly wages (Rs.)	Standard deviation of monthly wages (Rs.)
A	50	40 000	6 750
B	60	35 000	7 000

Determine which section has the larger variability in wages.

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5. The sum of the numbers and the sum of the squares of the numbers of a set of 20 observations are 140 and 2260 respectively.

(i) Find the mean and the standard deviation of the 20 observations.

(ii) If the median is 10, find the coefficient of skewness and comment on the shape of the distribution of the set of 20 observations.

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6. The probability that a randomly selected seed from a packet germinates is 0.7. If five seeds are randomly selected from the packet for planting, find the probability that

(i) at least one of the seeds will germinate,

(ii) exactly three seeds will germinate.

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7. A box has two red pens, two blue pens and a black pen. Two pens are randomly selected without replacement. Find the probability that both pens selected are of

(i) the same colour,

(ii) different colours.

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8. The probability mass function of a discrete random variable X is given below:

x	0	1	2	3
$P(X = x)$	0.2	0.2	0.3	0.3

Find $E(X)$.

Let Y be the random variable given by $Y = 2X - 3$. Find $E(Y)$ and the probability that Y is positive.

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[see page six

9. Suppose that A and B are exhaustive events of a sample space S. If $P(A) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{5}$, find (i) $P(B)$, (ii) $P(A|B)$, (iii) $P(A'|B')$.

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10. Let X be a continuous random variable with probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(3x-1), & 1 \le x \le 4, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

Find

(i) the value of k,

(ii) the mean of X.

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නව නිර්දේශය /புதிய பாடத்திட்டம்/New Syllabus

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය	II
கணிதம்	II
Mathematics	**II**

07 E II

Part B

* *Answer **five** questions only.*

11. A company owns two machines A and B which have different production capacities for high, medium and low grade nails. This company should produce at least 7, 6 and 13 tons per week of high, medium and low grade nails respectively to meet the demand of the market. It costs the company Rs. 10 000 and Rs. 8 000 per day to operate machines A and B respectively.

The following table gives the capacity of production in tons per day of each machine on each grade of nails.

Grade of nails	**Capacity (tons/day)**	
	A	***B***
High	2	1
Medium	1	1
Low	2	3

The company wishes to find out the number of days that each machine be operated per week to minimize the total production cost, while meeting the demand.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the solution of the problem formulated in part (i) above.

(iv) Due to a technical issue, the machine B has to be operated for a week at most twice as many days as the machine A is operated.

Find the increase of total production cost per week, if the company still wishes to minimize the production cost.

[See page eight

12. (*a*) Let $\mathbf{A} = \frac{1}{3}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{pmatrix}$.

Find x and y such that $\mathbf{AA}^{\mathrm{T}} = \mathbf{I}_3$, where $\mathbf{I}_3$ is the identity matrix of order 3 and $\mathbf{A}^{\mathrm{T}}$ represents the transpose of $\mathbf{A}$.

(*b*) Let $\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

Find constants p and q such that $\mathbf{A}^3 + p\mathbf{A} = q\mathbf{I}_3$, where $\mathbf{I}_3$ is the identity matrix of order 3.

Deduce that there is a square matrix $\mathbf{B}$ of order 3 such that $\mathbf{BA} = \mathbf{I}_3$.

Consider the following system of linear equations:

$$\begin{aligned} y + z &= 1 \\ x + z &= 2 \\ x + y &= 5 \end{aligned}$$

By taking $\mathbf{H} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ and $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, show that the matrix equation $\mathbf{AX} = \mathbf{H}$ represents the above system of linear equations.

Hence solve the above system of linear equations.

13. (*a*) Two unbiased standard six-sided dice I and II with faces marked 1, 2, 3, 4, 5, 6 are tossed. Let x and y be the numbers landed in die I and die II respectively.

Let A and B be the events defined by

$A : x \leq y$, and

$B : x + y$ is an odd integer.

Find $P(A)$, $P(B)$, $P(A \cap B)$ and $P(A \mid B)$.

(*b*) (i) Find the number of different permutations that can be formed from the ten letters of the word "STATISTICS".

(ii) Find the number of different combinations that can be formed from four letters taken from the ten letters of the word "STATISTICS".

|*See page nine*

14. Fruits are packed in three boxes, A, B and C, so that box A contains only 7 mangoes, box B contains 4 mangoes and 3 pears and box C contains 5 apples and 2 pears. Suppose that a box is selected at random and 2 fruits are randomly picked one after the other without replacement from the selected box.

Assuming that the selection of each box is equally likely, find the probability that

(i) both fruits selected are mangoes,

(ii) at least one of the selected fruits is a mango,

(iii) both fruits selected are mangoes given that one is a mango,

(iv) fruits are of different kinds.

15. A continuous random variable X has an exponential distribution with probability density function $f(x)$ given by

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & , \ x>0,\\ 0 & , \ \text{otherwise},\end{cases}$$

where $\lambda\ (>0)$ is a parameter.

Find the mean and the variance of X.

The lifetime X of an electric equipment is exponentially distributed with a mean of 2 years. Find the cumulative distribution function of X and **hence** find the median of X.
(You may take $e^{-0.7} \simeq 0.5$.)

An equipment is randomly selected. Find the probability that

(i) the life time of the equipment will exceed $1\frac{1}{2}$ years,

(ii) the equipment will fail before 2 years, given that the equipment had lasted more than $1\frac{1}{2}$ years.
(You need not simplify the answers.)

16. The mean and the standard deviation of the set of values $\{x_i : i=1,2,\ldots,n\}$ are μ and σ respectively. Find the mean and the standard deviation of the set of values $\{ax_i+b : i=1,2,\ldots,n\}$, where a and b are constants.

The following table summarises the ages (recorded to the nearest year) at the initial diagnosis of high blood sugar of a group of 70 diabetic patients.

Age	Number of patients
10 – 20	9
20 – 30	12
30 – 40	32
40 – 50	14
50 – 60	3

(i) Using a suitable linear transformation or otherwise, calculate the mean and the standard deviation of the given frequency distribution.

(ii) Find the inter-quartile range of the above distribution.

(iii) Two more patients who were both initially diagnosed with high blood sugar at the age of 55 joined the group. Find the inter-quartile range of the frequency distribution of the initial age of diagnosis of high blood sugar of all 72 patients.

[See page ten

17. Duration of activities in a project and the flow of activities are described in the following table:

Activity	Immediate predecessor(s)	Duration (in months)
A	–	2
B	A	2
C	A	3
D	B, C	4
E	B, D	5
F	–	8
G	E, F	1
H	E, G	2
I	H	4

(i) Construct the project network.

(ii) Prepare an activity schedule that includes earliest start time, earliest finish time, latest start time, latest finish time and float for each activity.

(iii) What are the activities that cannot be delayed without extending the total duration of the project?

(iv) Find the total duration of the project.

(v) Due to external reasons, activity F is expected to take one more month than the regular duration. Determine whether the project can still be completed within the total duration calculated in part (iv) above.

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