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ල් ලංකා විතාන දෙපාර්තමේන්තුව ල් ලංකා විතාන දෙපාර්ත**්න අවතාමේ ජනය ලෙපාර්තමේන්තුව**ාන දෙපාරතමේන්තුව ල් ලංකා විතාන දෙපාරතමේන්තුව இலங்கைப் பரீட்சைத் නිකානස්සභාව இலங்கைப் பரீட்சைத் නිකානස්සභාව இත්වෙන් වූ ලංකා විතාන දෙපාරතමේන්තුව Department of Examinations, Sri Lanka Department of the control of

> අධායන පොදු සහතික පතු (උසස් පෙළ) විභාගය, 2021(2022) සහ්බෝධ பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2021 (2022) General Certificate of Education (Adv. Level) Examination, 2021 (2022)

උසස් ගණිතය I உயர் கணிதம் I Higher Mathematics I



் පැය තුනයි மூன்று மணித்தியாலம் Three hours

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number)
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Instructions:

- * This question paper consists of two parts;
 Part A (Questions 1 10) and Part B (Questions 11 17).
- * Part A:

 Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- * Part B:

 Answer five questions only. Write your answers on the sheets provided.
- * At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.
- * You are permitted to remove only Part B of the question paper from the Examination Hall.

For Examiners' Use only

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		Part A	··
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2.	Let a relation R be defined of	on the set of all integers \mathbb{Z} by aRb if $5a + b$ is	divisible by 3. Show that
	R is an equivalence relation	on Z and write down the equivalence class o	f 0.
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3. Let $f(x) = \frac{x}{x-3}$ for $x \neq 3$ and $g^{-1}(x) = 2x-1$ for $x \in \mathbb{R}$.

Find $f^{-1}(x)$ and g(x), and show that $g(2f^{-1}(0)) = \frac{1}{2}$.

 	 : 	

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4. Show that

 $\begin{vmatrix} x^3 + x & x+1 & x-2 \\ 2x^3 + 3x + 1 & 3x & 3x - 3 \\ x^3 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = x \begin{vmatrix} 1 & 1 & 1 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 1 & -2 \\ -4 & 0 & 0 \\ 3 & -3 & 3 \end{vmatrix}$

5.	The chord through the Q to the pa	focus	of the	parabola	a. Show								
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6.	Let $f(x) = \begin{cases} \begin{cases} f(x) = \\ f(x) \end{cases} \end{cases}$	$\frac{\alpha x + \beta x - 1}{\beta x - 1}$	$\frac{ x }{ x }$ if	$x \neq 0$ $x = 0$	and	$g(x) = \begin{cases} \\ \end{cases}$	$\frac{\sqrt{1+\alpha}}{\alpha}$	$\frac{x-1}{x}$	if x	c≠0 ==0,			
	where $\alpha > 0$					`	•						
	It is given that	-		c) are co	ntinuou	s at $x = 0$). Find	the val	ues o	f α and	eta.		
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·.	Let $f(x) = \begin{cases} \begin{cases} f(x) = \\ f(x) \end{cases}$	$-x^2$,	if	-1< <i>x</i> <0,	
	Let $f(x) = {$	-x-2,	if	$x \le -1$.	a. Here is the second of the s
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ලි ලංකා විගාන දෙපාර්තමේන්තුව ලි ලංකා විභාග දෙපාර්තමේන්තුව කියලා සිදුවර්ගන්තුව සිදුවර්ගන්තුව ලි ලංකා විභාග දෙපාර්තමේන්තුව இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் படுக்கு இணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் Department of Examinations, Sri Lanka Department of **இலங்கை Sufficion த**ின்**ணிக்கனம்**ns, Sri Lanka Department of Examinations. Sri Lanka G ලංකා විභාග දෙපාර්තමේන්තුව ලී ලංකා විභාග සහ ප්රේක සහ

අධායන පොදු සහතික පතු (උසස් පෙළ) විභාගය, 2021(2022) கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2021(2022) General Certificate of Education (Adv. Level) Examination, 2021(2022)

උසස් ගණිතය I உயர் கணிதம் I Higher Mathematics I



Part B

- * Answer five questions only.
- 11.(a) Let A, B and C be subsets of a universal set S. Stating clearly the Laws of Algebra of sets that you use, show that
 - (i) $(A-B) \cup (A-C) = A (B \cap C)$,
 - (ii) $(A'-B)\cap C' = (A'-C)-(B-C)$,

where A - B is defined by $A \cap B'$.

- (b) In a group of Football, Basketball and Volleyball players, it is given that
 - (i) 8 players can play Football and Basketball,
 - (ii) 5 players can play Football and Volleyball,
 - (iii) 7 players can play Basketball and Volleyball,
 - (iv) 29 players can play Football or Basketball,
 - (v) 30 players can play Football or Volleyball and
 - (vi) 25 players can play Basketball or Volleyball

Find how many players can play Football.

- 12.(a) Let a, b, c > 0.
 - (i) Show that $ab \le \frac{1}{2}(a^2 + b^2)$ and **deduce** that $abc^2 \le \frac{1}{4}(a^4 + b^4 + 2c^4)$.
 - (ii) **Hence**, show that $abc \le \left(\frac{a^4 + b^4 + c^4}{a + b + c}\right)$. Show that the equality holds if and only if a = b = c.
 - (b) The transformation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 6 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ maps points in the xy-plane to the points in the x'y'-plane. Find the equations of the straight lines which are mapped onto them-selves. Find the image of the line y = 2x 1 in the x'y'-plane.

13. State and prove De Moivre's Theorem for a positive integral index.

$$\cos 4\theta = \cos^4 \theta - 6\cos^2 \theta \sin^2 \theta + \sin^4 \theta \text{ and}$$

$$\sin 4\theta = 4\cos^3 \theta \sin \theta - 4\cos \theta \sin^3 \theta.$$

Hence, show that
$$\cot 4\theta = \frac{\cot^4 \theta - 6\cot^2 \theta + 1}{4(\cot^3 \theta - \cot \theta)}$$
.

Solve
$$\cot 4\theta = \sqrt{3}$$
 and show that $x = \cot\left(\frac{\pi}{24}\right)$ is a solution of the equation $x^4 - 4\sqrt{3}x^3 - 6x^2 + 4\sqrt{3}x + 1 = 0$.

Write down the other solutions of this equation also in the form
$$\cot\left(\frac{k\pi}{24}\right)$$
 stating the values of k.

Deduce that
$$\cot \frac{\pi}{24} + \cot \frac{7\pi}{24} + \cot \frac{13\pi}{24} + \cot \frac{19\pi}{24} = 4\sqrt{3}$$
.

14.(a) Let C_1 and C_2 be the curves given by $y = (x-1)^2 + 1$ and $(y-2)^2 = 16x$ respectively. Sketch the graphs of C_1 and C_2 in the same diagram indicating the coordinates of their points of intersection. Find the area of the region R bounded by the curves C_1 and C_2 .

Also, find the volume of the solid generated by rotating the region R through 2π radians about the line y = 1.

(b) A family of curves is defined by the differential equation $\frac{dy}{dx} = \frac{2x - y + 5}{-x + 2y + 5}$.

Find the equation of the curve of the family that passes through the origin.

15.(a) Let
$$I_n = \int_0^1 x^n (1-x)^{\frac{3}{2}} dx$$
, for $n \in \mathbb{Z}^+$.

Show that,
$$I_n = \left(\frac{2n}{2n+5}\right)I_{n-1}$$
 for $n \ge 2$.

Hence, find the value of
$$\int_{0}^{1} x^{4} (1-x)^{\frac{3}{2}} dx$$

(b) Find the Maclaurin series of $\cos x$ and e^x in ascending powers of x up to and including the term in x^3 .

Hence, obtain the Maclaurin series of $e^{-x} \cos(x^2)$ in ascending powers of x up to and including the term in x^3 .

Using this, find an approximate value for
$$\int_{0}^{0.1} xe^{-x} \cos(x^2) dx.$$

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16. Show that the equation of the tangent at the point $(a\cos\theta, b\sin\theta)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $bx\cos\theta + ay\sin\theta = ab$ and the equation of the tangent at $(r\cos\phi, r\sin\phi)$ to the circle $x^2 + y^2 = r^2$ is $x\cos\phi + y\sin\phi = r$.

Let C be the circle $x^2 + y^2 = 36$ and S be the ellipse $\frac{x^2}{9^2} + \frac{y^2}{4^2} = 1$.

Suppose that $(6\cos\phi, 6\sin\phi)$ is a point of intersection of the circle C and the ellipse S. Show that $\tan^2\phi = \frac{4}{9}$.

Hence, or otherwise show that the acute angle between the tangents to the ellipse S and the circle C at the points of intersection is $\tan^{-1}\left(\frac{5}{9}\right)$.

- 17.(a) Let $f(x) = \frac{\cos^2 x}{2 + 2\sin x \cos x + \sin^2 x}$ for $x \in \mathbb{R}$.
 - (i) Show that $0 \le f(x) \le \frac{3}{5}$ for $x \in \mathbb{R}$.
 - (ii) Solve the equations $f(x) = \frac{3}{5}$ and f(x) = 0, and sketch the graph of y = f(x) for $0 \le x \le \frac{\pi}{2}$.
 - (b) The following table gives values of the function f(x) correct to two decimal places for values of x between 0 to 1.2 at intervals of length 0.2.

х	0.0	0.2	0.4	0.6	0.8	1.0	1.2
f(x)	1.12	2.01	0.00	1.11	1.65	2.42	1.61

Using Simpson's Rule, find an approximate value for $I = \int_{0}^{1.2} f(x) dx$.

Hence, find an approximate value for $\int_{0}^{1.2} x f'(x) dx.$

മ്മാള്യ മിയ്മായ് අවේරිණි /முழுப் பதிப்புரிமையுடையது /All Rights Reserved)

ල් ලංකා විභාග දෙපාර්තමේන්තුව ල් ලංකා විභාග දෙපාර්ත**ින්තුව කාලම්න්තුව ලෙපාර්තමේන්තුව**ාග දෙපාර්තමේන්තුව ල් ලංකා විභාග දෙපාර්තමේන්තුව இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் Department of Examinations, Sri Lanka Department of **ඔබෝබෝග් SAU සැබීම් කියාවේ පැරණිම් මේ**ම් ලංකා විභාග අදපාරතමේන්තුව ල් ලංකා විභාග අදපාරතමේන්තුව ල් ලංකා විභාග දෙපාරතමේන්තුව ල් ලංකා විභාග අද**ානවේන් අතිරිධ පැරණිව කියාවේ සිටියාවේ සිටියාවේ** ඉහතින් අවශ්ය දෙපාරතමේන්තුව ල් ලංකා විභාග අදපාරතමේන්තුව இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரீட்சைத் திணைக்களம்

අධානයන පොදු සහතික පනු (උසස් පෙළ) විභාගය, 2021(2022) සහ්බෝධ பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2021(2022) General Certificate of Education (Adv. Level) Examination, 2021(2022)

උසස් ගණිතය II உயர் கணிதம் II Higher Mathematics II



சැக තුනයි மூன்று மணித்தியாலம் **Three hours** Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

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Instructions:

* This question paper consists of two parts;

Part A (Questions 1 - 10) and Part B (Questions 11 - 17).

* Part A:

Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.

* Part B:

Answer five questions only. Write your answers on the sheets provided.

- * At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.
- * You are permitted to remove only Part B of the question paper from the Examination Hall.
- * Statistical Tables will be provided.
- * g denotes the acceleration due to gravity.

For Examiners' Use only

(11) Higher Mathematics II					
Part	Question No.	Marks			
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Part A

ı.	Let $A \equiv (-2, -1, -1)$, $B \equiv (3, 1, 2)$ and $C \equiv (1, \alpha)$		t the
	angle between \overrightarrow{OA} and \overrightarrow{OC} is $\frac{2\pi}{3}$ and $ \overrightarrow{OA} = \overrightarrow{OC} $	β . Find the values of α and β .	v.*
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		ven that their lines of action pass through the pe	
	$4\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$. Find the values of α , β and γ . Write resultant.		
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	$4\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$. Find the values of α , β and γ . Write resultant.		

3.	A uniform solid right circular cone of radius a and height $4a$ and of doint immersed in a homogeneous liquid of density ρ with its vertex at a dissurface of the liquid. Find the value of the ratio $\frac{\sigma}{\rho}$.	ensity on tance a	floats above	partially the free
		-		
	Find the smallest weight of the particle that can be attached to the verte it totally immersed in the liquid.	x of the	e cone	to make
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4.	The acceleration of a particle P at time t is given by $a(t) = 6t\mathbf{i} - \cos t\mathbf{j} + e^t\mathbf{k}$.	The po	sition v	ector and
4.		The po	sition v	ector and
4.	The acceleration of a particle P at time t is given by $a(t) = 6t\mathbf{i} - \cos t\mathbf{j} + e^t\mathbf{k}$. the velocity of the particle P at $t = 0$ are $\mathbf{j} + \mathbf{k}$ and \mathbf{k} respectively. Find the	The po	sition v	ector and
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5.	a smooth horizontal plane, each with speed u , towards on a smooth vertical wall which meets the wall in the line DE . They both hit the wall a the point X with Q arriving at X before P . The direction of motion of P is along P 0 along P 1. The direction of motion of P 2 is along P 2 and P 3 along P 3 along P 4. Where P 4 and P 5 along P 6 and P 6 and P 7 and P 8 and P 9 and the wall is P 9 and the	_	_	
	Show that after rebounding from the wall, the speeds of P and Q are in the ra	itio √.	5:1.	
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6.	A uniform rod OA of mass m and length $2a$ is free to rotate in a vertical plane about the end O which is fixed. It is held in a position making an angle $\frac{2\pi}{3}$ with the downward vertical and then released. Show that when the rod makes an angle θ with the downward vertical the angular speed $\dot{\theta}$ satisfies $a\dot{\theta}^2 = \frac{3}{4}g(1+2\cos\theta)$.	O	2a	
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find the probability that,	
(i) exactly one customer,	
ii) less than 3 customers,	
order tea with their breakfast.	
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	$f(x) = \begin{cases} \frac{1}{c} \\ 0 \end{cases}$	x^2 ,	<i>x</i> ≤1				
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An un	biased cubic	die is rolle		be the value	obtained and	$\int_{0}^{\infty} \int_{0}^{\infty} dt Y = \frac{1}{2} X^{2} .$	
An un	biased cubic	die is rolle		be the value	obtained and	$\int_{0}^{\infty} \int_{0}^{\infty} dt Y = \frac{1}{2} X^{2} .$	
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An un	biased cubic	die is rolle		be the value	obtained and	$\int_{0}^{\infty} \int_{0}^{\infty} dt Y = \frac{1}{2} X^{2} .$	
An un	biased cubic	die is rolle	ed once. Let A	be the value	obtained and	$\int_{0}^{\infty} \int_{0}^{\infty} dt Y = \frac{1}{2} X^{2} .$	

සිගලුම හිමිකම් ඇවිරිණි/ ψ ගුව යනිව්ටුගිකාගයුණ $All\ Rights\ Reserved$

ලි ලංකා විශාත දෙපාර්තමේන්තුව ලී ලංකා විශාන දෙපාර්තල්න්තුව කිරීම ප්රධාන සිදුවල් ප්රධාන දෙපාර්තලේන්තුව ලී ලංකා විශාන දෙපාර්තමේන්තුව මූහාකිකසට පුර්ධානෙන් නිකාන්යයහාර මූහක්කසට පුර්ධාන මූහක්කත්ව ප්රධානය ප්රධානයේ සහ මූහක්කයා ප්රධානයේ නිකාන්යයහාර මූ Department of Examinations, Sri Lanka Department of **මූහාකියන්**වා ප්රධානයේ සහ ප්රධානය

අධායන පොදු සහතික පතු (උසස් පෙළ) විහාගය, 2021(2022) සහ්බෝධ ධොළාුුු தூநதரப் பத்திர (உயர் தூ)ப் பர்ட்சை, 2021(2022) General Certificate of Education (Adv. Level) Examination, 2021(2022)

උසස් ගණිතය II உயர் கணிதம் II Higher Mathematics II



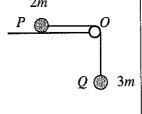
Part B

- * Answer five questions only.
- 11. Let $A \equiv (1, 0, 0)$, $B \equiv (0, 1, 0)$ and $C \equiv (0, 0, 1)$ be three points. Forces, $\overrightarrow{2AB}$, $\overrightarrow{3AC}$ and \overrightarrow{BC} act along AB, AC and BC in the sense indicated by the letters respectively.
 - (i) Show that the system, reduces to a single force \mathbf{R} acting through the point A and a couple \mathbf{G} , where \mathbf{R} and \mathbf{G} are to be determined.
 - (ii) Now a force F is introduced to the above system.
 - (a) If **F** acts through the origin and the system reduces to a couple, find **F** and the magnitude of the couple.
 - (b) If **F** acts at the point with the position vector $\mathbf{i} + c\mathbf{j} + d\mathbf{k}$ and the system is in equilibrium, find the values of c and d.
- 12. A circular lamina of radius a is immersed with its surface vertical in a homogeneous liquid of constant density ρ such that its centre O at a depth a below the free surface of the liquid. Show that the magnitude of the liquid thrust on the lamina is $\pi a^3 \rho g$, and that the centre of pressure of the lamina lies on the vertical diameter, at a distance $\frac{a}{4}$ below the centre O.

A solid right circular cone of radius a and height 2a is immersed in a homogeneous liquid of constant density ρ , with its highest point on the free surface of the liquid and its axis horizontal. Find the magnitude, direction and the line of action of the thrust on the curved surface of the cone.

13. A particle P of mass 2m placed on a rough horizontal table is connected to a particle Q of mass 3m by a light inextensible string which passes over a small smooth pulley O fixed at the edge of the table. Particles P and Q and the string all lie in a vertical plane.

Let the coefficient of friction between P and the table be $\frac{1}{2}$. The system is released from rest with the string taut, as shown in the diagram. Show that the particles start to move.

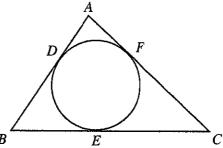


The particle Q moves in a resistive medium that offers a resistance mkv when its speed is v, where k(>0) is a constant. Show that $5\frac{dv}{dt} = 2g - kv$.

Find the time taken by P to reach a speed of $\frac{g}{k}$ and the distance travelled by P during this period. (Assume that the length of the string is long enough for P not to reach O during this period.)

- 14. Two smooth spheres, A and B, of equal radius, lie on a smooth horizontal table. A is of mass m and B is of mass 2m. The spheres are projected towards each other and when they colide, the line joining their centers is parallel to j and velocities of A and B are 2i + 3j and $-i + \alpha j$ respectively, where $\alpha > 0$. The coefficient of restitution between A and B is $\frac{1}{2}$ and the velocity of the sphere Q just after the collision is -i + 3j. Find
 - (i) the value of α
 - (ii) the velocity of the sphere P just after the collision
 - (iii) the loss in kinetic energy caused by the collision
 - (iv) the impulse on A from B.
- 15. A frame consists of three uniform rods, each of mass m and length 2a, rigidly joined together to form the triangle ABC, together with a uniform circular ring of mass m inscribed in the triangle, as shown in the figure. The ring is rigidly fixed to the rods AB, BC and CA at D, E and F respectively, where D, E and F are mid-points of AB, BC and CA respectively.

Show that the moment of inertia of the frame about the axis through A perpendicular to the plane of the frame is $\frac{23}{3}ma^2$.



The frame can rotate about a fixed smooth horizontal axis through A, the axis being perpendicular to the plane of the frame. The frame is given a small displacement from the equilibrium position in which the center of mass of the frame is below A, and released from rest. Show that the motion of the frame is approximately simple harmonic and its period is $2\pi \sqrt{\frac{23a}{8\sqrt{3}g}}$.

16. (a) Suppose the discrete random variable X has the probability distribution given below:

X	0	1	2	3	4
P(X=x)	0.1	0.3	0.4	0.15	0.05

Let Y = 2X + 1. The probability distribution of Y is given by the following table.

Y	1	3	5	7	9
P(Y=y)	0.1	0.3	p	q	0.05

- (i) Find the values of p and q.
- (ii) Find E(Y) and Var(Y).
- (iii) Find P(Y > 3), and hence find P(X > 1).

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- (b) (i) Let X be the number of tails obtained when 3 unbiased coins are tossed. Find the probability distribution of X and hence find E(X) and Var(X).
 - (ii) If the value of X is an odd number, a biased cubical die with the probability of getting 3 or 6 on the face, equals to $\frac{2}{3}$ is rolled. Otherwise, another biased cubical die with the probability of getting 3 or 6 on the face, equals to $\frac{1}{3}$ is rolled. Variable Y is defined as follows:

$$Y = \begin{cases} 2, & \text{if the value on the face of the die is divisible by 3} \\ 1, & \text{otherwise} \end{cases}$$

Find the probability distribution of Y and hence, find E(Y) and Var(Y).

17.(a) The probability density function of the continuous random variable X is given by

$$f_X(x) = \begin{cases} 10x^2(1-x) & , & 0 < x < 1 \\ 0 & , & \text{otherwise} \end{cases}$$

Find E(X) and Var(X).

Also, find $P(X < \frac{1}{2})$.

If the continuous random variable Y is defined by $Y = \frac{3X+2}{4}$, find E(Y) and Var(Y).

- (b) Suppose that daily household expenditures on transport in an urban area are normally distributed with a mean of Rs. 2000 and a standard deviation of Rs. 400.
 - (i) Find the probability that the daily transport cost of a randomly selected household exceeds Rs. 2500.
 - (ii) It is given that the daily transport cost of 10% of the households exceeds Rs. k. Find the value of k.