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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2017 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2017 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2017

උසස් ගණිතය I
உயர் கணிதம் I
**Higher Mathematics I**

**11 E I**

පැය තුනයි
மூன்று மணித்தியாலம்
*Three hours*

| **Index Number** | | | | | | | |
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## Instructions:

* *This question paper consists of two parts;*
  **Part A** (Questions 1 – 10) and **Part B** (Questions 11 – 17).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

**For Examiners' Use only**

**(11) Higher Mathematics I**

| Part | Question No. | Marks |
|---|---|---|
| A | 1 | |
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|---|---|
| Paper I | |
| Paper II | |
| Total | |
| Final Marks | |

**Final Marks**

| | |
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| In Words | |

**Code Numbers**

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| Marking Examiner | | |
| Checked by: | 1 | |
| | 2 | |
| Supervised by: | | |



[see page two

## Part A

1. Factorize: $(x+y+z)^5-x^5-y^5-z^5$.

2. A relation $R$ is defined on the set of all positive real numbers $\mathbb{R}^+$ by $xRy$ if $\sin^2 x+\cos^2 y = 1$. Show that $R$ is an equivalence relation on $\mathbb{R}^+$. Find the equivalence class of $\pi$.



[see page three

**3.** Let $a,\ b \in \mathbb{R}^+$ with $a \neq 0$, and let $f(x) = ax+b$ for $x \in \mathbb{R}$. Show that $f$ is one-to-one and onto. Let $g(x) = 2x+1$ for $x \in \mathbb{R}$. Show that if $f \circ g = g \circ f$, then $a = b+1$.

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**4.** Show that $\begin{vmatrix} 1 & \cos^2\alpha & \sin^2\alpha \\ 1 & \cos^4\alpha & \sin^4\alpha \\ 1 & \sec^2\alpha & \tan^2\alpha \end{vmatrix} = 2\sin^4\alpha\cos^2\alpha$.

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[see page four

**5.** Show that the equation of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(x_1, y_1)$ is $\frac{x_1x}{a^2}+\frac{y_1y}{b^2}=1$.

Deduce the equation of the tangent to the auxiliary circle $x^2+y^2=a^2$, at the point $(x_1, y_2)$. Show that these two tangents meet at a point on the major axis of the ellipse, provided that $x_1 \neq 0$ and $y_1 \neq 0$.

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**6.** Let $a>0$ and $b>0$, and let $f: \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x)=\begin{cases} \frac{|x+a|}{x+a} & \text{if} \quad x>-a \\ 2x+1 & \text{if} \quad -a \leq x \leq b \\ 2+\ln(x+1-b) & \text{if} \quad x>b \end{cases}$$

It is given that $f$ is continuous on $\mathbb{R}$. Find the values of $a$ and $b$.

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[see page five

7. Let $f:\mathbb{R}\to\mathbb{R}$ be the function defined by $f(x)=x|x|$ for $x\in\mathbb{R}$. Show that $f(x)$ is differentiable at $x=0$ and write down $f'(x)$ for all $x\in\mathbb{R}$.
Show that $f'(x)$ is **not** differentiable at $x=0$.

8. Solve the differential equation $\frac{dy}{dx}+\frac{2}{x}y=\sqrt{x^3+3}$, subject to the condition $y=2$ when $x=1$.



[see page six

**9.** Let $a \in \mathbb{R}$ and let $f$ be a real-valued continuous function defined on $[0, 2a]$ such that $f(x + a) = f(x)$ for $x \in [0, a]$. Show that $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$.

**10.** Sketch the curves whose polar equations are given by $r = 2 \cos \theta$ and $r = 2 \sin \theta$ in the same diagram and find the area of the region bounded by these two curves.

General Certificate of Education (Adv. Level) Examination, August 2017

Higher Mathematics I

11 E I

**Part B**

* Answer **five** questions only.

**11.** (*a*) Let $A$, $B$, $C$ and $D$ be subsets of a universal set $S$. Stating clearly the Laws of Algebra of Sets that you use, prove each of the following:

(i) $(A \cap B \cap C \cap D)' = (A' \cup B' \cup C' \cup D')$

(ii) $(A-B)-C = (A-C)-(B-C)$, where $A-B$ is defined by $A-B = A \cap B'$.

(*b*) A survey of 600 students was conducted to determine which sports they like from among cricket, volleyball and football. The following data were collected from the survey:

206 like cricket, 141 like volleyball, 184 like football. Also 42 like cricket and volleyball, 65 like cricket and football, 57 like volleyball and football, and 19 like all three sports.

Of the sports surveyed, find the number of students who like

(i) at most one sport,

(ii) exactly two sports,

(iii) only football.

Also, find the number of students who do not like any of the sports.

**12.** (*a*) The Arithmetic mean-Geometric mean Inequality for three positive real numbers $a$, $b$ and $c$ is given by

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}\,.$$

(i) When does the equality hold here?

(ii) Find all pairs of $a$ and $b$ such that $a^3+b^3=3ab$.

(iii) Show that $(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2) \geq 9a^2b^2c^2$.

(iv) What is the minimum value of $a+\dfrac{1}{b(a-b)}$, when $a > b$?

(*b*) The transformation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, maps the points in the $xy$-plane into $x'y'$-plane. Show that if $ad-bc \neq 0$, the parallel lines in the $xy$-plane are mapped onto parallel lines in the $x'y'$-plane by this transformation.



[see page eight

**14.** Two small smooth spheres $P$ and $Q$ of the same radius and of the same mass, with centres $A$ and $B$ respectively, are moving towards each other on a smooth horizontal floor. Just before collision, the velocity $\mathbf{u}$ of $P$ makes an acute angle $\theta$ with $\overrightarrow{AB}$, and the velocity $\mathbf{v}$ of $Q$ is along $\overrightarrow{BA}$. The coefficient of restitution between the two spheres is $e$. Find the components of velocity of $P$ along and perpendicular to the line of centres $AB$, just after impact.

Show that the sphere $Q$ continues to move in the same direction as before, with speed $(1-e)\frac{v}{2}-\frac{(1+e)}{2}u\cos\theta$, provided that $u<\left(\frac{1-e}{1+e}\right)v$, where $v=|\mathbf{v}|$ and $u=|\mathbf{u}|$.

Also, show that if $u\cos\theta \ll v$, kinetic energy retained in $Q$ is a fraction $\frac{1}{4}(1-e)^2$ of its original value.

**15.** A uniform solid sphere of mass $M$ and radius $r$ is released from rest on a fixed rough plane of inclination $\alpha$ to the horizontal. The coefficient of friction between the sphere and the plane is $\mu$.

(i) Show that, if $\mu>\frac{2}{7}\tan\alpha$, the sphere will roll down the plane and its centre will have a constant acceleration $a$, given by $a=\frac{5}{7}g\sin\alpha$.

(ii) Show that if $\mu<\frac{2}{7}\tan\alpha$, the sphere will slide down the plane, and the acceleration of its centre is greater than $a$.

(iii) Show further that if $\mu=\frac{2}{7}\tan\alpha$ and if the centre of the sphere is initially given a velocity $u$ along a line of greatest slope down the plane, without rotating the sphere, that velocity will remain unchanged.

[It may be assumed that the moment of inertia of a uniform solid sphere of mass $M$ and radius $r$ about a diameter is $\frac{2}{5}Mr^2$.]

**16.** (*a*) For a discrete random variable $X$, define the mean $E(X)=\mu$ and obtain the formula $\mathrm{Var}(X)=E(X^2)-\mu^2$ for the variance of $X$.

(*b*) The probability distribution of a discrete random variable $X$ is as follows:

| $x$ | 1 | 2 | 4 | 5 |
|---|---|---|---|---|
| $P(X=x)$ | $p$ | $q$ | $q$ | $p$ |

Given that $p=\frac{1}{12}$, find the value of $q$.

Show that $E(X)=3$, and find $\mathrm{Var}(X)$.

The random variable $Y$ is defined by $Y=X_1+X_2$, where $X_1$ and $X_2$ are two independent observations of $X$. Show that $P(Y=6)=\frac{13}{36}$, and obtain the probability distribution of $Y$.

Find $E(Y)$ and $\mathrm{Var}(Y)$.

**Verify** that $E(Y)=2E(X)$ and that $\mathrm{Var}(Y)=2\,\mathrm{Var}(X)$.



[see page nine

**17.** (*a*) A continuous random variable $X$ has probability density function $f(x)$ given by

$$f(x)=\begin{cases} kx(1-x) & , \quad \text{if } 0 \le x \le 1, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

(i) Show that $k=6$.

(ii) Find $P\left(X>\frac{1}{2}\right)$.

(iii) Find $E(X)$ and Var(X).

(*b*) The weights of bags of tea are normally distributed with mean 200 g. It is given that exactly 60% of all tea bags have weights between 190 g and 210 g.

(i) Find the standard deviation of the weights of the tea bags.

(ii) Find the probability that a randomly chosen tea bag has a weight between 180 g and 200 g.

(iii) Four tea bags are randomly chosen. Find the probability that at least one of these bags has a weight more than 210 g.

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**இலங்கைப் பரீட்சைத் திணைக்களம்**
**Department of Examinations, Sri Lanka**

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2017 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தரப் பரீட்சை, 2017 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2017

උසස් ගණිතය II
உயர் கணிதம் II
**Higher Mathematics II**

**11 E II**

පැය තුනයි
மூன்று மணித்தியாலம்
***Three hours***

| **Index Number** | | | | | | | |
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**Instructions:**

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* *You are permitted to remove **only*** **Part B** *of the question paper from the Examination Hall.*
* *Statistical Tables will be provided.*
* *g denotes the acceleration due to gravity.*

**For Examiners' Use only**

| (11) Higher Mathematics II | | |
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| **Part** | **Question No.** | **Marks** |
| A | 1 | |
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| Supervised by: | | |



[see page two

**Part A**

1. The position vectors of three points $A$, $B$ and $C$ with respect to an origin $O$ are $\mathbf{i}$, $2\mathbf{j}$ and $2\mathbf{k}$ respectively. By considering the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$ show that

   (i) the area of the triangle $ABC$ is $\sqrt{6}$ square units, and

   (ii) a unit vector perpendicular to the plane of $ABC$ is $\dfrac{2\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{6}}$.

2. A force $\mathbf{F}$ of magnitude 15 N acts at the point with position vector $a\mathbf{i} + b\mathbf{j}$ in the direction of the vector $2\mathbf{i}-\mathbf{j}+2\mathbf{k}$. Find the values of constants $a$ and $b$ measured in metres, if the moment vector of $\mathbf{F}$ about the origin $O$ is $10\mathbf{i}+20\mathbf{j}$ N m.



[see page three

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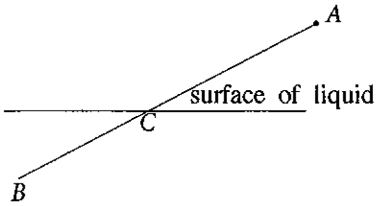
**3.** A uniform rod $AB$ of length $2a$ and density $\rho$ smoothly hinged at the end $A$ is in equilibrium in an inclined position with the part $BC$ of length $2b$ immersed in a homogeneous liquid of density $\sigma$, as shown in the figure. Show that $\frac{\rho}{\sigma} = \frac{2ab - b^2}{a^2 + 2ab - b^2}$.

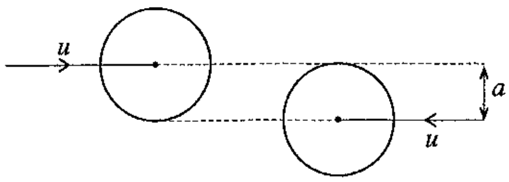



**4.** The position vector of particle $P$ of mass $m$ relative to a fixed origin $O$, at time $t$ is $\mathbf{r}$. It is given that the force $\mathbf{F}$ acting on $P$ is directed towards $O$. Show that its angular momentum vector $\mathbf{h}$ defined by $\mathbf{h} = \mathbf{r} \times m\mathbf{v}$ remains constant, where $\mathbf{v}$ is the velocity.
Also, show that the path of the particle lies on a plane whose equation may be expressed in the form $\mathbf{r} \cdot \mathbf{h}$ = constant.



[see page four

5. Two equal smooth spheres, each of radius $a$, are moving towards each other, with the same speed $u$ on a smooth horizontal floor, in opposite directions along two parallel lines whose distance apart is $a$. The coefficient of restitution between them is $\frac{1}{3}$. Show that, after their impact, each sphere moves with speed $\frac{u}{\sqrt{3}}$ perpendicular to the original direction of its motion.




6. The end $A$ of a uniform rod $AB$ of mass $m$ and length $2a$ is smoothly hinged to a fixed point. While the rod is hanging in equilibrium it is given an angular speed $\omega$. Show that, if $\omega^2 \geq \frac{3g}{a}$, the end $B$ will describe a complete circle.



[see page five

7. Let $X$ be the random variable "the number of heads obtained when four fair coins are tossed". Find the expectation of $X$ and show that the variance of $X$ is 1.

8. 10% of the items produced by a machine are defective. Find the probability that out of 5 items chosen from the production at random, at most 2 items will be defective.



[see page six

**9.** The probability density function $f(x)$ of a random variable $X$ is given by

$$f(x)=\begin{cases} a(2-x) & \text{if} \quad 1 \le x \le 2, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Show that (i) $a = 2$, (ii) the mean, $\mu = \frac{4}{3}$ , and (iii) $P(1 \le X \le 1.5) = 0.75$.

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**10.** The probability density function, $f(x)$, of a continuous random variable $X$ is given by

$$f(x)=\begin{cases} \frac{1}{10}e^{-\frac{x}{10}} & \text{if} \quad x \ge 0 \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Show that $P(X \le x) = 1 - e^{-\frac{x}{10}}$.

**Hence**, find $P(5 < X \le 10)$.

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தரப்) பரீட்சை, 2017 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2017

උසස් ගණිතය II
உயர் கணிதம் II
**Higher Mathematics II**

**11 E II**

## Part B

* Answer **five** questions only.

**11.** Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act at the points with the position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively as specified below:

| Point of action | Force |
|---|---|
| $\mathbf{r}_1 = 2\mathbf{i} - 4\mathbf{j}$ | $\mathbf{F}_1 = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$ |
| $\mathbf{r}_2 = -3\mathbf{j} + 5\mathbf{k}$ | $\mathbf{F}_2 = -\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ |
| $\mathbf{r}_3 = 3\mathbf{i} - \mathbf{k}$ | $\mathbf{F}_3 = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ |

Show that this system of forces is equivalent to a single force $\mathbf{R}$, together with a couple of moment vector $\mathbf{G} = 4\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$, when reduced at the origin $O$.

**Hence**, show that the system reduces to a single resultant force.

Find the magnitude of $\mathbf{R}$ and obtain a vector equation for the line of action of the resultant force, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{R}$, where $\lambda$ is a parameter and $\mathbf{a}$ is the position vector of a point to be determined.

**12.** A semicircular lamina of centre $O$ and radius $a$ is immersed in a homogeneous liquid with its plane vertical and the diameter on the free surface of the liquid. Using integration, find the liquid thrust on the lamina and show that the centre of pressure of the lamina is at a depth $\frac{3\pi}{16}a$ from $O$.

A door in the shape of a semicircle of centre $O$ and radius $a$ is made on the vertical side of a tank. The door is smoothly hinged along the diameter $AB$ which is horizontal and the door lies below $AB$. The tank is filled to the level of $AB$ with a homogeneous liquid of density $\rho$. Find the least force that should be applied to the door to keep it closed, so that the liquid is inside the tank.

**13.** A particle of mass $m$ is projected horizontally with initial speed $u$ on a smooth horizontal floor. The resistance to its motion is $\lambda m v^{\frac{3}{2}}$, where $\lambda$ is a positive constant and $v$ is the speed of the particle at time $t$. Show that $\frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$ and **hence**, obtain the relation $v = \frac{4u}{(2 + \lambda\sqrt{u}\,t)^2}$.

Show further that the time taken by the particle for the speed to reduce from $u$ to $\frac{u}{4}$ is $\frac{2}{\lambda\sqrt{u}}$ and find the distance travelled by the particle during this time period.



[see page eight

**14.** Two small smooth spheres $P$ and $Q$ of the same radius and of the same mass, with centres $A$ and $B$ respectively, are moving towards each other on a smooth horizontal floor. Just before collision, the velocity $\mathbf{u}$ of $P$ makes an acute angle $\theta$ with $\overrightarrow{AB}$, and the velocity $\mathbf{v}$ of $Q$ is along $\overrightarrow{BA}$. The coefficient of restitution between the two spheres is $e$. Find the components of velocity of $P$ along and perpendicular to the line of centres $AB$, just after impact.

Show that the sphere $Q$ continues to move in the same direction as before, with speed $(1-e)\frac{v}{2}-\frac{(1+e)}{2}u\cos\theta$, provided that $u<\left(\frac{1-e}{1+e}\right)v$, where $v=|\mathbf{v}|$ and $u=|\mathbf{u}|$.

Also, show that if $u\cos\theta \ll v$, kinetic energy retained in $Q$ is a fraction $\frac{1}{4}(1-e)^2$ of its original value.

**15.** A uniform solid sphere of mass $M$ and radius $r$ is released from rest on a fixed rough plane of inclination $\alpha$ to the horizontal. The coefficient of friction between the sphere and the plane is $\mu$.

(i) Show that, if $\mu>\frac{2}{7}\tan\alpha$, the sphere will roll down the plane and its centre will have a constant acceleration $a$, given by $a=\frac{5}{7}g\sin\alpha$.

(ii) Show that if $\mu<\frac{2}{7}\tan\alpha$, the sphere will slide down the plane, and the acceleration of its centre is greater than $a$.

(iii) Show further that if $\mu=\frac{2}{7}\tan\alpha$ and if the centre of the sphere is initially given a velocity $u$ along a line of greatest slope down the plane, without rotating the sphere, that velocity will remain unchanged.

[It may be assumed that the moment of inertia of a uniform solid sphere of mass $M$ and radius $r$ about a diameter is $\frac{2}{5}Mr^2$.]

**16.** (*a*) For a discrete random variable $X$, define the mean $E(X)=\mu$ and obtain the formula $\text{Var}(X)=E(X^2)-\mu^2$ for the variance of $X$.

(*b*) The probability distribution of a discrete random variable $X$ is as follows:

| $x$ | 1 | 2 | 4 | 5 |
|---|---|---|---|---|
| $P(X=x)$ | $p$ | $q$ | $q$ | $p$ |

Given that $p=\frac{1}{12}$, find the value of $q$.

Show that $E(X)=3$, and find $\text{Var}(X)$.

The random variable $Y$ is defined by $Y=X_1+X_2$, where $X_1$ and $X_2$ are two independent observations of $X$. Show that $P(Y=6)=\frac{13}{36}$, and obtain the probability distribution of $Y$.

Find $E(Y)$ and $\text{Var}(Y)$.

**Verify** that $E(Y)=2E(X)$ and that $\text{Var}(Y)=2\,\text{Var}(X)$.



[*see page nine*

**17.** (*a*) A continuous random variable $X$ has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(1-x) & , \quad \text{if } 0 \le x \le 1, \\ 0 & , \quad \text{otherwise.} \end{cases}$$

(i) Show that $k = 6$.

(ii) Find $P\left(X > \frac{1}{2}\right)$.

(iii) Find $E(X)$ and $\text{Var}(X)$.

(*b*) The weights of bags of tea are normally distributed with mean 200 g. It is given that exactly 60% of all tea bags have weights between 190 g and 210 g.

(i) Find the standard deviation of the weights of the tea bags.

(ii) Find the probability that a randomly chosen tea bag has a weight between 180 g and 200 g.

(iii) Four tea bags are randomly chosen. Find the probability that at least one of these bags has a weight more than 210 g.

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