Confidential

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கல்வி அமைச்சு
Ministry of Education

G.C.E.(A.L) Support Seminar - 2023

10 – Combined Mathematics - I

Marking Scheme

1. Using the **principal of Mathematical induction** prove that $\sum_{r=1}^{n} 2^r = 2(2^n - 1)$ for all $n \in \mathbb{Z}^+$

$When\ n = 1 \quad L.H.S.\ 2^r = 2 \qquad R.H.S.\ 2(2^1 - 1) = 2$

$L.H.S. = R.H.S.$

$\therefore It\ is\ true\ for\ n = 1$ (05)

$Assume\ that\ it\ is\ true\ for\ n = k, k \in \mathbb{Z}^+$

$$\sum_{r=1}^{k} 2^r = 2(2^k - 1)$$ (05)

$$when\ n = k+1 \quad \sum_{r=1}^{k+1} 2^r = \sum_{r=1}^{k} 2^r + 2^{k+1}$$ (05)

$$= 2(2^k - 1) + 2^{k+1}$$

$$= 2(2^{k+1} - 1)$$ (05)

$\therefore the\ result\ is\ true\ for\ n = k+1$

$By\ principal\ of\ mathematical\ induction\ The\ result\ is\ true\ for\ all\ n \in \mathbb{Z}^+$ (05)

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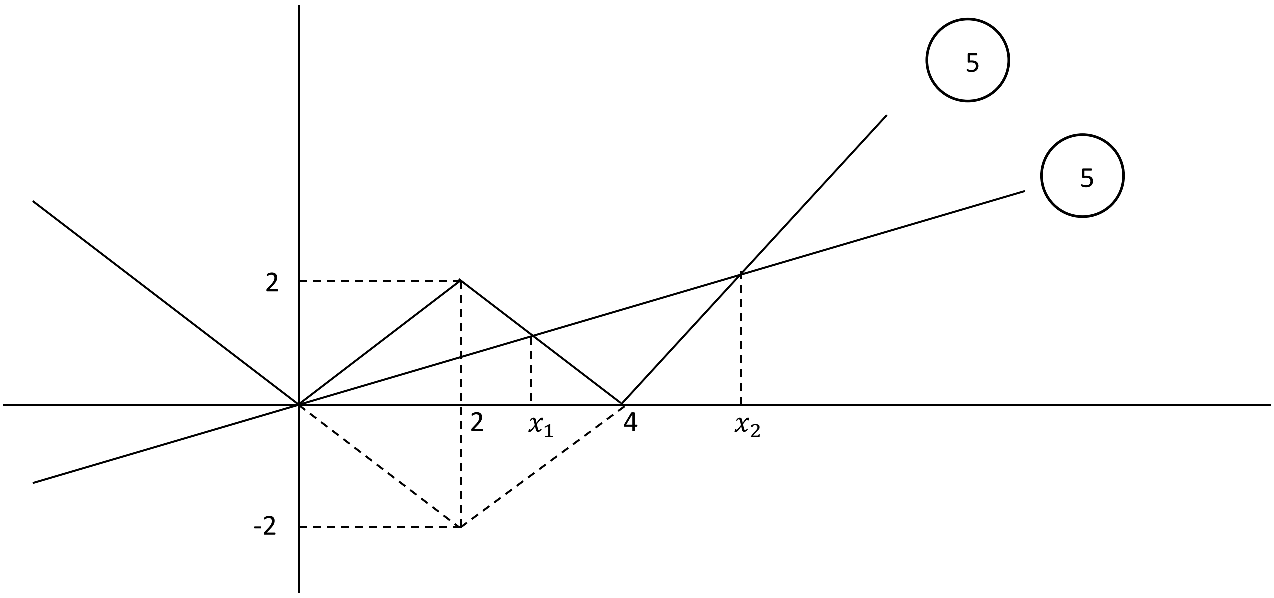
2 Sketch the graph of $= \left||x-2|-2\right|$. Hence or otherwise solve the equation $\left||x-2|-2\right| = \frac{x}{2}$



$$-x_1 + 4 = \frac{x_1}{2}$$

$$-2x_1 + 8 = x_1$$

$$x_1 = \frac{8}{3}$$ (5)

$$-x_2 + 4 = \frac{x_1}{2}$$

$$2x_2 - 8 = x_2$$

$$x_2 = 8$$ (5)

$$x = 0 \ OR \ x = \frac{8}{3} \ OR \ x = 8$$ (5)

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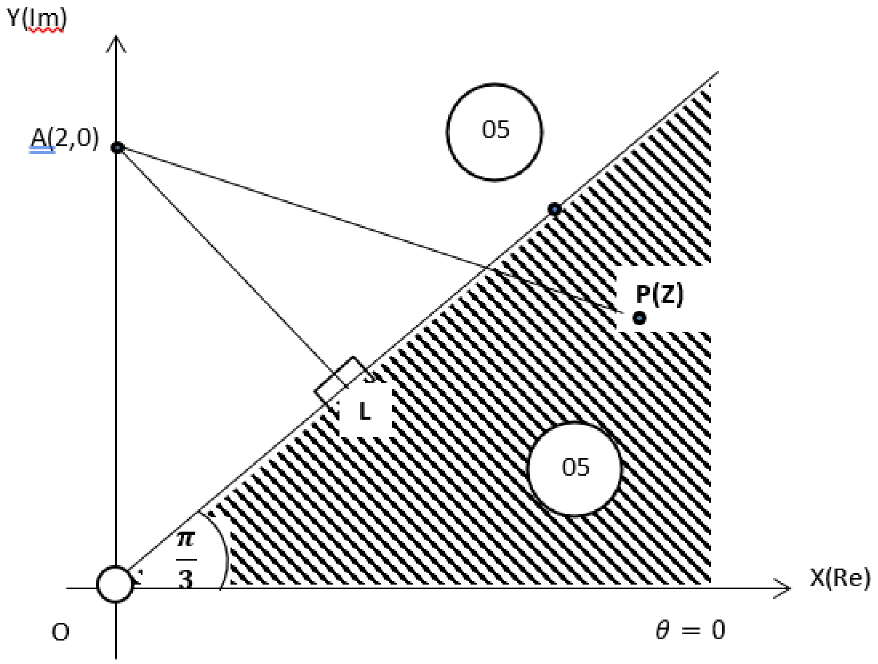
3 Shade the region R that represents the complex number Z satisfying the condition $0 \le ArgZ \le \frac{\pi}{3}$ in an Argand Diagram.

Also find the least Value of $|iZ + 2|$ in the region R.

$0 \le ArgZ \le \frac{\pi}{3}$



$|iZ + 2| = |i(Z - 2i)| = |Z - 2i|$ (05)

$= AP$

$\therefore |iZ + 2|_{least} = |Z - 2i|_{least} = AP_{least}$

$= AL = 2\sin\frac{\pi}{6}$ (05)

$= 1\ unit$ (05)

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4 **Write down** the binomial expansion of $(1+x)^n$ in ascending powers of x. Given that the coefficient of x^2 in the expansion $(1+x+ax^2)^7$ is 14. Show that $a=-1$.

$n, r \in \mathbb{Z}^+ \ n \geq r \quad {}^nC_r = \frac{n!}{r!(n-r)!}$ (05)

$(1+x)^n = = {}^nC_0\, x^0 + {}^nC_1\, x^1 + {}^nC_2\, x^2 + \ldots\ldots + {}^nC_r\, x^r + \ldots\ldots + {}^nC_n\, x^n$ (05)

$(1+x)^n = \sum_{r=0}^{n} \text{nCr}\, x^r$

$(1+x+ax^2)^7 \qquad = [1+(x+ax^2)]^7$

$= {}^7C_0 + {}^7C_1\,(x+ax^2) + {}^7C_2\,(x+ax^2)^2 + \ldots\ldots\ldots$ (05)

$= \ldots\ldots\ldots + (7a+21)\, x^2 + \ldots\ldots\ldots$

Coefficient of $x^2 = 14$ (05)

$\Rightarrow 7a + 21 = 14$ (05)

$a = -1$

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5 Show that $\lim_{x \to \frac{\pi}{4}} \frac{2\sqrt{x}-\sqrt{\pi}}{\sin(x-\frac{\pi}{4})} = \frac{2}{\sqrt{\pi}}$

$$\lim_{x\to\frac{\pi}{4}} \frac{2\sqrt{x}-\sqrt{\pi}}{\sin\left(x-\frac{\pi}{4}\right)} = \frac{2}{\sqrt{\pi}}$$

$$\frac{(2\sqrt{x}-\sqrt{\pi})}{\sin\left(x-\frac{\pi}{4}\right)} \times \frac{(2\sqrt{x}+\sqrt{\pi})}{(2\sqrt{x}+\sqrt{\pi})}$$ (05)

$$\lim_{x\to\frac{\pi}{4}} \frac{4x-\pi}{\sin\left(x-\frac{\pi}{4}\right)} \times \lim_{x\to\frac{\pi}{4}} \frac{1}{(2\sqrt{x}+\sqrt{\pi})}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{4x-\pi}{\sin\left(x-\frac{\pi}{4}\right)} \times \lim_{x\to\frac{\pi}{4}} \frac{1}{(2\sqrt{x}+\sqrt{\pi})}$$ (05)

$$4 \times \frac{1}{\lim_{\left(x-\frac{\pi}{4}\right)\to 0} \frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)}} \times \lim_{x\to\frac{\pi}{4}} \frac{1}{(2\sqrt{x}+\sqrt{\pi})}$$ (05)

$$4 \times \frac{1}{1} \times \frac{1}{2\sqrt{\pi}} = \frac{2}{\sqrt{\pi}}$$ (05) (05)

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6 If $f(x) = (x+1)tan^{-1}\sqrt{x} - \sqrt{x}$, then find $\frac{d[f(x)]}{dx}$. Hence, deduce $\int tan^{-1}\sqrt{x}\, dx$. The region enclosed by the curves $y = \sqrt{tan^{-1}\sqrt{x}}$, $x = 3$ and $y = 0$ is rotated about the $x-$ axis through 2π radians. Show that the volume of the solid thus generated is $\frac{\pi}{3}(4\pi - 3\sqrt{3})$ cubic units.

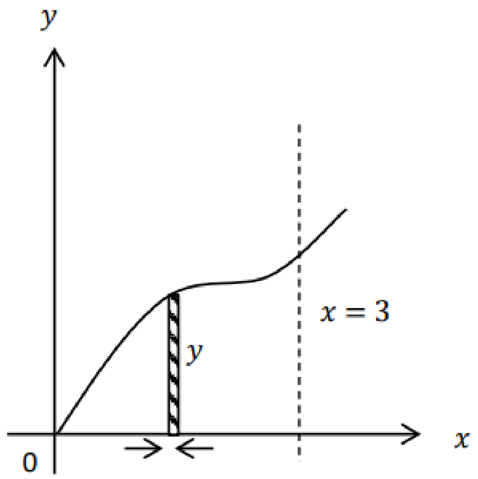
$\frac{d}{dx}\left[(x+1)tan^{-1}\sqrt{x} - \sqrt{x}\right] = (x+1)\frac{1}{(1+x)2\sqrt{x}} + tan^{-1}\sqrt{x} - \frac{1}{2\sqrt{x}}$ 05

$= tan^{-1}\sqrt{x}$ //

$\Rightarrow \int tan^{-1}\sqrt{x}\, dx = (x+1)tan^{-1}\sqrt{x} - \sqrt{x} + c$ 05

$x = 0, y = \sqrt{tan^{-1}\sqrt{0}}$

$= 0$



$V = \int_0^3 \pi\, y^2 dx = \pi \int_0^3 tan^{-1}\sqrt{x}\, dx$ 05

$= \pi\left[(x+1)tan^{-1}\sqrt{x} - \sqrt{x}\right]^3$

$= \pi\left[4\,tan^{-1}\sqrt{3} - \sqrt{3} - 0\right]^0$ 05

$= \pi\left(4\,{}^{\pi}/_{3} - \sqrt{3}\right)$

$= \pi\left(4\pi - 3\sqrt{3}\right)$ Cubic units 05

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7 A curve C is given by the parametric equations $x = a\cos\theta$ and $y = b\sin\theta$ for $(0 \le \theta \le \pi)$. Show that the equation of the normal to the curve C, at point P, is $ax\sec\alpha - by\operatorname{cosec}\alpha + b^2 - a^2 = 0$.

Also find the normal to the curve C, at point $\left(-\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ on the curve C.

$x = a\ \cos\theta,\ y = b\ \sin\theta$

$$\frac{dx}{d\theta} = -a\ \sin\theta$$

$$\frac{dy}{d\theta} = b\cos\theta$$

$$\frac{dy}{dx} = \frac{-b\cos\theta}{a\ \sin\theta} = \frac{-b}{a}cot\theta \quad (05)$$

Gradient of the normal $\frac{a}{b}tan\theta = \frac{a}{b}tan\alpha$ (05)

∴ Equation, $y - bsin\alpha = \frac{atan\alpha}{b}(x - acos\alpha) = \frac{asin\alpha}{bcos\alpha}(x - acos\alpha)$

$bycos\alpha - b^2sin\alpha cos\alpha = axsin\alpha - a^2sin\alpha cos\alpha$

$axsin\alpha - bycos\alpha - (a^2 - b^2)sin\alpha cos\alpha = 0$

$axsec\alpha - bycosec\alpha + b^2 - a^2 = 0$ (05)

$\frac{-a}{\sqrt{2}} = acos\theta$ $\qquad$ $\frac{b}{\sqrt{2}} = bsin\theta$

$cos\theta = -\frac{1}{\sqrt{2}} =$ $\qquad$ $sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{3\pi}{4}$ $\qquad$ $\theta = \frac{3\pi}{4}$ (05)

$-\sqrt{2}ax - \sqrt{2}by + b^2 - a^2 = 0$

$\sqrt{2}ax + \sqrt{2}by + a^2 - b^2 = 0$ (05)

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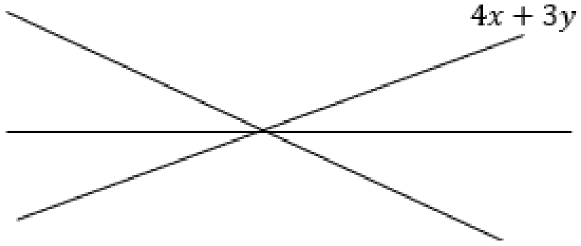
8 The straight line $l \equiv y - mx = 0$ passes through the point of intersection of two straight lines $4x + 3y - k = 0$, where k is constant and $5x - 12y + 7 = 0$. Find the value of m in terms of k. Further, given that the line, $l = 0$ is perpendicular to the line $x + y = 0$. Find the values of m and k.



$4x + 3y - k + \lambda(5x - 12y + 7) = 0$ (05)

$(4 + 5\lambda)x + (3 - 12\lambda)y - k + 7y = 0 \;\; c \Rightarrow y - mx = 0$

$m = \frac{4+5\lambda}{12\lambda-3}$ (05) $\qquad -k + 7\lambda = 0$ (05)

$\lambda = +\frac{k}{7}$

(05)

$$m = \frac{4 + 5\left(\frac{k}{7}\right)}{12\left(\frac{k}{7}\right) - 3} = \frac{28 + 5k}{12k - 21}$$

$(\because x + y = 0 \perp)$

$$m = 1 = \frac{28 + 5k}{12k - 21} \Leftrightarrow 12k - 21 = 28 + 5k$$

$7k = 49$

$K = 7$ (05)

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9 A circle S with centre on the y-axis intersects the circle $x^2 + y^2 = 9$ orthogonally and the circle $x^2 + y^2 + x - 7y + 5 = 0$ bisects the circumference of the circle S. Show that there are two such circle S, and find their equations.

$Lets\ S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$g = 0\ (\because Centre\ lies\ on\ y\ axis)$ (05)

$x^2 + y^2 - 9 = 0$

$g = 0, f = 0, c_1 = -9$

$2g(0) + 2f(0) = c - 9 \Rightarrow c = 9$ (05)

$x^2 + y^2 + 2fy + 9 = 0$

$x^2 + y^2 + x - 7y + 5 = 0$

Common chord $x - 7y - 2fy - 4 = 0$ (05)

$0 + 7f + 2f^2 - 4 = 0$

$2f^2 + 7f - 4 = 0$

$(2f - 1)(f + 4) = 0$

$f = \frac{1}{2}\ or\ f = -4$ (05)

$\therefore equation\ is\quad x^2 + y^2 + y + 9 = 0$

$x^2 + y^2 - 8y + 9 = 0$ (05)

$\because x + y = 0\ line\ in\ perpendicular\ to\ the\ line\ l = 0$

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10 Given that $tanA = \frac{5}{12}$ and $sinB = \frac{4}{5}$; Where A and B are such that $\pi < A < \frac{3\pi}{2}$ and $\frac{\pi}{2} < B < \pi$.

Find the value of $sin(A + B)$

$tan^2A + 1 = sec^2A$

$cosA = \pm\frac{1}{\sqrt{tan^2A - 1}}$

$= \pm\frac{1}{\sqrt{\frac{1}{1+\frac{25}{144}}}}$

$= \pm\frac{12}{13}$

$\because \pi < A < \frac{3\pi}{2}$

$cosA = -\frac{12}{13}$ (05)

$cosB = \pm\sqrt{1 - sin^2A}$

$\pm\sqrt{1 - \frac{16}{25}}$

$\pm\frac{3}{5}$

$\because \frac{\pi}{2} < A < \pi$

$cosB = -\frac{3}{5}$ (05)

$sinA = \pm\sqrt{1 - cos^2A}$

$= \pm\sqrt{1 - \frac{144}{169}}$

$= \pm\frac{5}{13}$

$\because \pi < A < \frac{3\pi}{2}$; $sinA = -\frac{5}{13}$ (05)

$sin(A + B) = sinAcosB + cosAsinB$

$= \left(-\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(-\frac{12}{13}\right)\left(\frac{4}{5}\right)$ (05)

$= \frac{15}{65} - \frac{48}{65}$

$= -\frac{39}{65}$ (05)

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Part B

11 (a) Write down the sum and the product of the roots of quadratic equation $ax^2 + bx + c = 0$, in terms of a, b and c where $a, b, c \in \mathbb{R}, a \neq 0$

Given that $f(x) \equiv x^2 - p^2qx + q^2$ where $p, q \in \mathbb{R}^+$ and roots of the equation $f(x) = 0$ are α and β.

(i) Find $\alpha^{\frac{3}{2}} + \beta^{\frac{3}{2}}$ in terms of p and q.

(ii) If α and β are real, then find the least integer value of p.

(iii) For the above p value find the quadratic equation in terms of q, whose roots are $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$.

(b) Let $P(x) \equiv 2x^3 + x^2 - 2x + \lambda$; where $\lambda \in \mathbb{R}^+$

(i) If λ is zero of the polynomial $P(x)$, find λ

(ii) If $-\lambda$ is zero of the polynomial $P(x)$, find λ

(iii) For the value of λ which satisfies both (i) and (ii), write down the polynomial $P(x)$ and express $P(x)$ as a multiple of linear factors.

(iv) Find the remainder, when $P(x) + 3x + 2$ is divided by $x^2 + 1$.

$ax^2 + bx + c = 0$ roots are $\propto, \beta$

$\propto + \beta = -\frac{b}{a}$ (05) $\propto\beta = \frac{c}{a}$ (05)

$f(x) = 0, x^2 - p^2qx + q^2 = 0$ $\propto, \beta$

$\propto + \beta = p^2q$ (05) $\propto\beta = q^2$ (05)

$\left(\propto^{\frac{3}{2}} + \beta^{\frac{3}{2}}\right)^2 = \alpha^3 + \beta^3 + 2\alpha^{\frac{3}{2}}\beta^{\frac{3}{2}}$ (05)

$= (\propto + \beta)(\propto 2 + \beta 2 \quad \propto\beta) + 2\sqrt{\propto^3 \beta^3}$ (05)

$= (\propto + \beta)[(\propto + \beta)^2 - 3\propto\beta] + 2\sqrt{(\propto \beta)^3}$ (05)

$= p^2q\,(p^4q^2 - 3q^2) + 2\,q^3$

$= q^3\,[p^2\,(p^4 - 3) + 2]$ (05)

$\propto^{\frac{3}{2}} + \beta^{\frac{3}{2}} = \sqrt{q^3[p^2(p^4 - 3) + 2]}$

40

Real roots for $\propto, \beta$ $\Delta x \geq 0$ (05)

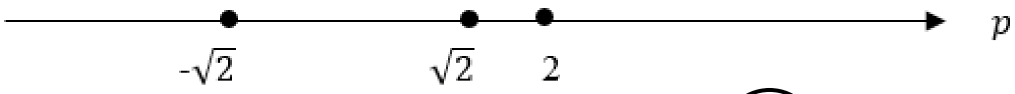
$$p^4 q^2 - 4q^2 \geq 0$$ (05)

$q^2 > 0 \quad p^4 - 4 \geq 0$

$(p2 - 2)\,(p2 + 2) \geq 0$ (05)

$\left(p - \sqrt{2}\right)\,\left(p + \sqrt{2}\right) \geq 0$

$(p - \sqrt{2})\,(p + \sqrt{2}) \geq 0$ (05)



least integer of p is 2 (05)

Sum of two roots

$\propto^{\frac{3}{2}} + \beta^{\frac{3}{2}} = \sqrt{q^2\,(p^2(p^4\,3\,) + 2}$

$P = 2$

$= \sqrt{q^3(4\,(16 - 3) + 2}$

$= \sqrt{54\,q^3}$ (05)

Product of two roots

$= \sqrt{\propto^3\ \beta^3} = \sqrt{(q^2\,)^3} \qquad = q^3$ (05)

Quadratic Equation $>$ $\quad x^2 - \sqrt{54\,q^3}\ x + q^3 = 0$ (05)

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(b)

(i)

$f(x) \equiv 2x^3 + x^2 - 2x + \lambda$

λ *is Zero for a polynomial* $f(x), f(\lambda) = 0$ 05

$2\lambda^3 + \lambda^2 - 2\lambda + \lambda = 0$ 05

$\lambda(2\lambda^2 + \lambda - 1) = 0$

$(2\lambda - 1)(\lambda + 1) = 0 \ (\because \ \lambda \neq 0)$ 05

20

$\lambda = \frac{1}{2}$ *or* $\lambda = -1$ 05

(ii)

$(-\lambda)$ *is a zero of* $f(x), f(-\lambda) = 0$ 05

$-2\lambda^3 + \lambda^2 + 2\lambda + \lambda = 0$

$-2\lambda^3 + \lambda^2 + 3\lambda = 0$

$(\lambda \neq 0) \quad -\lambda(2\lambda^2 - \lambda - 3) = 0$

$(2\lambda - 3)(\lambda + 1) = 0$ 05

15

$\lambda = \frac{3}{2}$ *or* $\lambda = -1$ 05

(iii)

From (i) and (ii)

when $\lambda = -1$ 05

$f(x) \equiv 2x^3 + x^2 - 2x - 1$

10

$\equiv (x - 1)(x + 1)(2x + 1)$ 05

(iv)

$g(x) \equiv f(x) + 3x + 2$

$\equiv (x-1)(x+1)(2x+1) + 3x + 2$

$\equiv (x^2+1)(2x+a) + px + q$

$x = 1 \qquad 5 = 2(2+a) + p + q$

$x = -1 \qquad -1 = 2(-2+a) - p + q$

$x = 0 \qquad -1 + 2 = a + q$

$1 = 2a + p + q$ ——— (1)

$3 = 2a - p + q$ ——— (2)

$1 = a + q$ ——— (3)

(2) - (1) $\qquad p = -1$

(1) + (2) $\qquad q = 0$

$\therefore$ *when* $g(x)$ *is divided by* $x^2 + 1$, *the reminder is* $-x$. (05)

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12 (a) An institution has 8 cars and there are parking facilities in two rows, 4 cars in each row in the park.

(i) Find the number of ways in which 8 cars can be parked.

(ii) Find the number of ways in which 8 cars can be parked, if the first place in the first row is to be reserved for chairman's car and a place in the first row for the car of secretaries.

(iii) If the first place in the first row should be given to either one of the two cars of chairman's or secretaries, and if the other car should also have a place in the first row, find the number of ways in which the cars can be parked.

(b) Find the value of the constants A and B such that,

$$\frac{r^2+3r-1}{(r^2-r+1)(r^2+r+1)} = \frac{Ar+B}{r^2-r+1} - \frac{Ar+2B}{r^2+r+1} \quad ; \text{ where } r\epsilon\mathbb{Z}^+.$$

If, $U_r = \dfrac{r^2+3r-1}{(r^2-r+1)(r^2+r+1)}$ then determine f_r such that $U_r = f_r - f_{r+1}$

Hence, show that $\displaystyle\sum_{r=1}^{n} U_r = 2 - \frac{(n+2)}{n^2+n+1}$

Is this series convergent? Justify your answer.

If, $\displaystyle\sum_{r=1}^{n} U_r < 2 - \frac{11}{91}$ then find greatest value of n.

(a)

I. $^8C_4 \times 4! \times {}^4C_4 \times 4! = \frac{8\times7\times6\times5}{4!} \times 4! \times 4! = 8!$ (10)

$= 40320$ (05)

II. $^6C_2 \times 3! \times {}^4C_4 \times 4! = \frac{6\times5}{2\times1} \times 3\times2\times1\times1\times24$ (15)

$=15\times6\times24$

$=2160$ (05)

III. $^2C_1 \times {}^6C_2 \times 3! \times 4! \times {}^4C_4$ (20)

$=2 \times \frac{6\times5}{1\times2} \times 6 \times 24 \times 1$

$=2 \times 15 \times 6 \times 24$

$= 4320$ (05)

60

(b)

$$\frac{r^2+3r-1}{(r^2-r+1)(r^2+r+1)} = \frac{Ar+B}{(r^2-r+1)} - \frac{Ar+2B}{(r^2+r+1)}$$

$$r^2+3r-1 = (Ar+B)(r^2+r+1) - (Ar+2B)(r^2-r+1)$$

$$= Ar^3 + Ar^2 + Ar + Br^2 + Br + B - (Ar^3 - Ar^2 + Ar + 2Br^2 - 2Br + 2B$$

$$r^2+3r-1 = 2Ar^2 - Br^2 + 3Br - B$$

Comparing coefficients

$r^2 \to 2A - B = 1$ (05)

$r \to 3B = 3 \implies B = 1$ (05)

coefficients

$A = 1$ (05)

Constant satisfied $-B = -1$

$$u_r = \frac{r+1}{r^2-r+1} - \frac{r+2}{r^2+r+1}$$

$u_r = f_r - f_{r+1}$; $f_r = \frac{r+1}{r^2-r+1}$ (10)

$u_1 = f_1 - f_2$

$u_2 = f_2 - f_3$ (05)

$u_3 = f_3 - f_4$ +

$u_{n-1} = f_{n-1} - f_n$ (05)

$u_n = f_n - f_{n+1}$

$$\sum_{r=1}^{n} u_r = f_1 - f_{n+1}$$ (05)

$$\sum_{r=1}^{n} u_r = 2 - \frac{n+2}{n^2+n+1}$$ (05)

This series is convergent

$$\lim_{n\to\infty}\sum_{r=1}^{n} u_r = \lim_{n\to\infty} 2-\frac{n+2}{n^2+n+1}$$ (05)

$= 2-0 = 2\epsilon\mathbb{R}$ (05)

$$\sum_{r=1}^{n} u_r < 2-\frac{11}{91}$$ (05)

$$2-\frac{n+2}{n^2+n+1} < 2-\frac{11}{91}$$ (05)

$$\frac{n+2}{n^2+n+1} > \frac{11}{91}\ ;\ \ n>1$$

$91(n+2) > 11(n^2+n+1)$

$91n+182 > 11n^2+11n+11$

$0 > 11n^2-80n-171$ (05)

$0 > 11n^2-99n+19n-171$

$0 > 11n(n-9)+19(n-9)$

$0 > \underbrace{(11n+19)}_{>0}(n-9)\ \ ;n\in\mathbb{Z}^+$ (05) (05)

$0 > n-9$

$9 > n$ (05)

Thus maximum value of $n = 8$ (05)

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13 (a) If $A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, show that any real matrix B which commutates with A, under multiplication, can be written in the form $\lambda A + \mu I$, where λ *and* μ are real numbers and I is the identity matrix of order 2. Find the value of λ *and* μ *when* $B = A^2$ Hence Find A^{-1}.

(b) By Factorizing Z^6-1, completely solve the equation $Z^6 = 1$.

If Z_1 and Z_2 are any two distinct roots of the equation $Z^6 = 1$, show by reference to an Argent diagram, or otherwise, that the three possible values of $|Z_1 - Z_2|$ are 1, 2 and $\sqrt{3}$.

(c) By using De moivre's theorem for positive integer n,

Show that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos n\left(\frac{\pi}{2}-\theta\right) + i\sin n\left(\frac{\pi}{2}-\theta\right)$

Deduce that, $\left(\frac{1+i}{1-i}\right)^{2n} = (-1)^n$

(a) Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (05)

$\Leftrightarrow \begin{bmatrix} 2a+b & a \\ 2c+d & c \end{bmatrix} = \begin{bmatrix} 2a+c & 2b+d \\ a & b \end{bmatrix}$ (05)

$\Leftrightarrow 2a+b = 2a+c$

$a + 2b + d$

$2c + d = a$

$c = b$ (15)

$\Leftrightarrow a = 2b+d, c = b$ (05)

$\therefore B = \begin{bmatrix} 2b+d & b \\ b & d \end{bmatrix} = \begin{bmatrix} 2b & b \\ b & 0 \end{bmatrix} + \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}$ (05)

$= b\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \lambda A + \mu I$ where $\lambda = b$ *and* $\mu = d$ (05)

$A^2 = AA = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ (05)

$B^2 = A^2 \Leftrightarrow \begin{bmatrix} 2\lambda+\mu & \lambda \\ \lambda & \mu \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ (05)

$\therefore \lambda = 2$ and $\mu = 1$ (05)

$\therefore A^2 = 2A + I \quad c.e.\ A^2 - 2A = I$

$A(A - 2I) = I$ (05)

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b) $Z^6 - 6 = 0$

$\Rightarrow (Z^3 - 1)(Z^3 + 1) = 0$ (05)

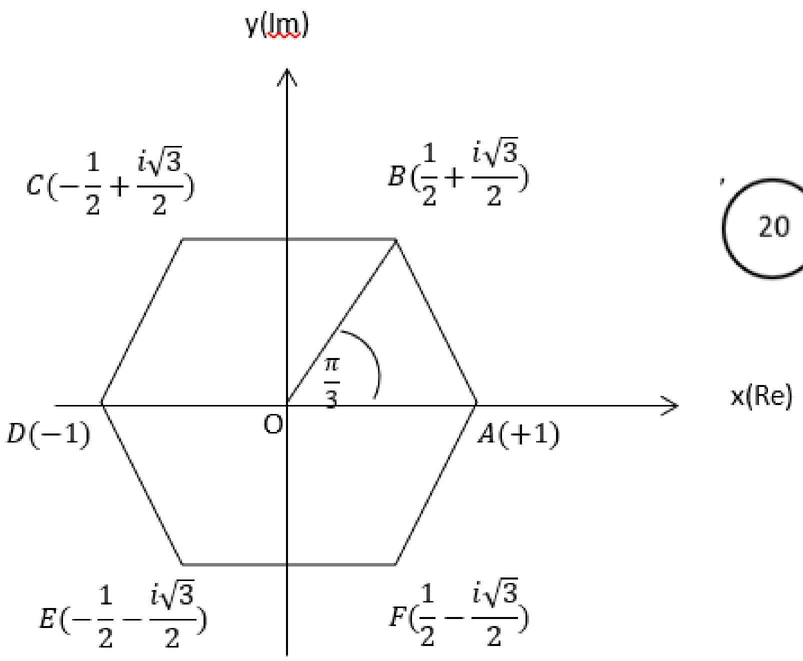
$(Z - 1)(Z^2 + Z = 1)(Z + 1)(Z^2 - Z + 1) = 0$

$Z = 1, \quad \frac{-1 \pm \sqrt{1-4}}{2}, \quad \frac{1 \pm \sqrt{1-4}}{2};$ (20)

This gives the six results

$Z = \pm 1, \frac{-1 \pm i\sqrt{3}}{2}, \frac{1 \pm i\sqrt{3}}{2}$; $i^2 = -1$

The modules and argument of each root are 1 and a multiple of $\frac{\pi}{3}$ respectively. On the Argand diagram there six roots can be represented by six points as shown in the figure.



(20)

$OA = OB = OC = OD = OE = OF = 1$ All the six points A, B, C, D, E, F lie on the circle with centre O and radius 1 unit. (05)

For any two of the six results Z_1 and Z_2

$|Z_1 - Z_2|$ is the length of the segment which joints any two points out of those six points. (10)

$\therefore |Z_1 - Z_2| = 1$ or 2 or $\sqrt{3}$ units

$\because AB = 1, AD = 2, AC = \sqrt{3}$

Alt: Consider the possible values of $|Z_1 - Z_2|$ algebraically.

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(c) $\left(\frac{1+sin\theta+icos\theta}{1+sin\theta-icos\theta}\right)^n = \left(\frac{sin^2\theta-i^2cos^2\theta+sin\theta+icos\theta}{1+sin\theta-icos\theta}\right)^n$ (05)

$$= \left(\frac{(sin\theta+icos\theta)(sin\theta-icos\theta)+(sin\theta+icos\theta)}{1+sin\theta-icos\theta}\right)^n$$

$$= \left(\frac{(sin\theta + icos\theta)(sin\theta - icos\theta + 1)}{1 + sin\theta - icos\theta}\right)^n$$ (05)

$$= (sin\theta + icos\theta)^n$$

$$= \left\{cos\left(\frac{\pi}{2} - \theta\right) + isin\left(\frac{\pi}{2} - \theta\right)\right\}^n$$ (05)

$= cos\ n\left(\frac{\pi}{2} - \theta\right) + isin\ n\left(\frac{\pi}{2} - \theta\right)$ (De Movers')

When $\theta = 0$ and replaced n by $2n$ for the above (05)

$\left(\frac{1+sin\theta+icos\theta}{1+sin\theta-icos\theta}\right)^{2n} = cos\ 2n\left(\frac{\pi}{2} - \theta\right) + i\ sin\ 2n\left(\frac{\pi}{2} - \theta\right)$ (05)

$= cos\ n\pi + i\ sin\ n\pi$

Where $sin2n\pi = 0$ and $cosn\pi \begin{cases} = +1\ ;\ n\ even \\ = -1\ ;\ n\ odd \end{cases}$ (05)

$\therefore \left[\frac{1+i}{1-i}\right]^{2n} = (-1)^n$

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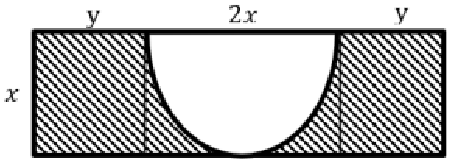
14 (a) Let $f(x) = \frac{x(x+3)}{(x+1)^2}$ for $x \neq -1$

Show that $f'(x)$ the first derivative of $f(x)$ with relative to x, is given by
$f'(x) = -\frac{(x-3)}{(x+1)^3}$

.

Hence, find the intervals on which $f(x)$ is decreasing and the intervals on which $f(x)$ is increasing.
Obtain the coordinates of the turning point of $f(x)$.
It is given that $f''(x) = \frac{2(x-5)}{(x+1)^4}$ for $x \neq -1$.
Find the coordinates of the point of inflection on the graph of $y = f(x)$.
Sketch the graph of $y = f(x)$ indicating the asymptotes, turning point and point of inflection.

(b) The shaded region shown in the figure is obtained by removing a semicircular lamina of radius $x\,m$ from a rectangle of length $2(x+y)\,m$ and width $x\,m$.



The area of the rectangle is $8\pi\,m^2$. Show that the perimeter p of the shaded region, measured in meters, is given by $P = \pi\left(x + \frac{16}{x}\right)$

(a)

$$f(x) = \frac{x(x+3)}{(x+1)^2}$$

$$f^1(x) = \frac{(x+1)^2(2x+3) - x(x+3)2(x+1)}{(x+1)^4}$$ (20)

$$= \frac{(x+1)(2x+3) - 2x(x+3)}{(x+1)^3}$$

$$= \frac{2x^2 + 5x + 3 - 2x^2 - 6x}{(x+1)^3}$$

$$= \frac{-x+3}{(x+1)^3}$$

$$f^1(x) = \frac{-(x-3)}{(x+1)^3}$$ (05)

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Turning point $\rightarrow f^1(x) = 0$ $\quad x = 3$ (05)

$y = \frac{18}{16} = \frac{9}{8}$ $\quad \left[3, \frac{9}{8}\right]$ (05)

Vertical asymptote $\rightarrow x = -1$ (05)

Horizontal asymptote $\rightarrow$ $---= \frac{x(x+3)}{(x+1)^2} = 1 \rightarrow y = 1$ (05)

	$-\alpha < x < -1$	$-1 < x < 3$	$3 < x < +\alpha$
$f^1(x)$ sign	< 0	> 0	< 0
Nature of the function	decrease	increase	decrease

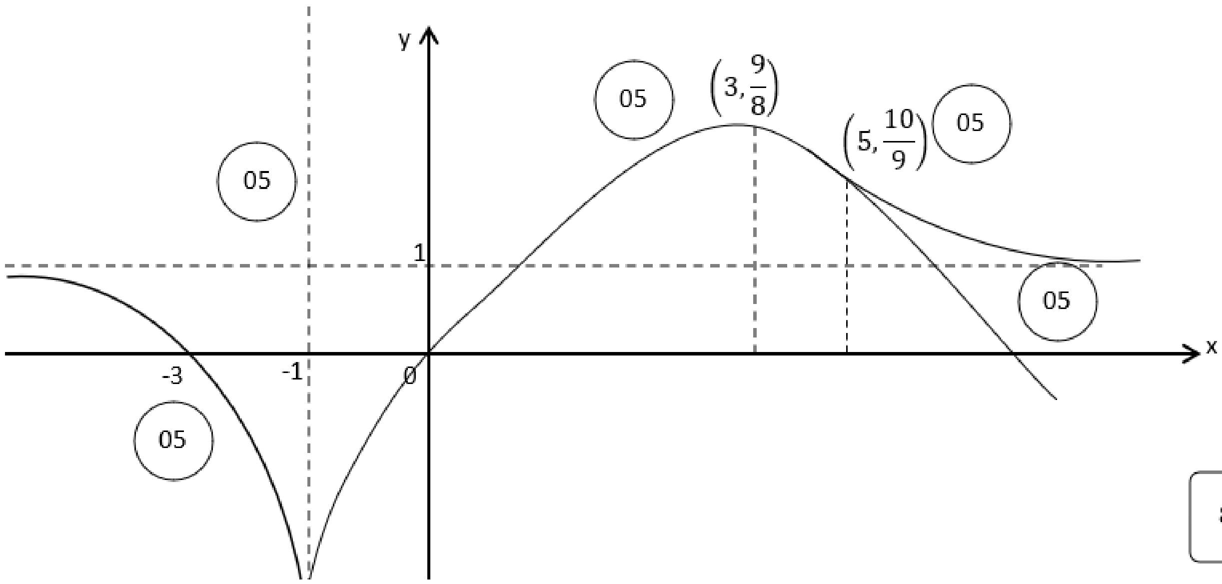
(15) (05)

Point of inflection$\rightarrow f^{11}(x) = 0 \rightarrow x = 5$ (05)

	$-\alpha < x < 5$	$5 < x < +\alpha$
$f^{11}(x)$ ලකුණ	< 0	> 0
Concavity	down ∩	up ∪

(05) (05)

$x = 5$ is the point of inflection



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(b)

$(2x + 2y)x = 8\pi$ (05)

$(x + y)x = 4\pi$

$x^2 + xy = 4\pi$

$y = \frac{4\pi - x^2}{x} = \frac{4\pi}{x} - x$ (05)

$P = 2x + 2x + 4y + \pi x = (4 + \pi)x + 4\left[\frac{4\pi}{x} - x\right]$ (05)

$P = 4x + \pi x + \frac{16\pi}{x} - 4x = \pi\left[x + \frac{16}{x}\right]$ (05)

$\frac{dP}{dx} = \pi\left[1 - \frac{16}{x^2}\right]$ (05)

$\frac{dP}{dx} = 0 \rightarrow x = 4\ (x > 0)$ (05)

	$x < 4$	$4 < x$
$\frac{dP}{dx}$	$-$	$+$
	(05)	(05)

$\therefore$ *when* $x = 4$, P is minimum (05)

45

15 (a) Determine the values of constants A, B and C such that

$x^4 + 1 = A(x^4 - 1) + B(x^2 + 1)(x + 1) + C(x^2 + 1)(x - 1) - (x^2 - 1)$ for $x \in \mathbb{R}$,

hence, find $\int \frac{x^4+1}{x^4-1} dx$

(b) (i) If $y = x + \cos x \sin^3 x$ show that $\frac{dy}{dx} = 1 + 3\sin^2 x - 4\sin^4 x$.

Given that $I = \int_0^{\frac{\pi}{2}} (x + 3x \sin^2 x - 4x \sin^4 x)\, dx$. By using above result and using integration by parts, Show that $I = \frac{1}{8}(\pi^2 - 2)$

(ii) Further given that,

$J_1 = \int_0^{\frac{\pi}{2}} (1 + 3\cos^2 x - 4\cos^4 x)\, dx$

$J_2 = \int_0^{\frac{\pi}{2}} (x + 3x \cos^2 x - 4x \cos^4 x)\, dx$

Using the result $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

Show that $I = \frac{\pi}{2} J_1 - J_2$

Now given that $\frac{d}{dx}(x - \sin x \cos^3 x) = 1 + 2\cos^2 x - 4\cos^4 x$

show that $J_2 = \frac{1}{8}(\pi^2 + 2)$, deduce the value of J_1.

(c) Using the substitution $\sqrt{x^3 + 1} = t$, Evaluate $\int_0^2 \frac{x^8}{\sqrt{x^3+1}} dx$.

(a) $x^4 + 1 = A(x^4 - 1) + B(x^2 + 1)(x + 1) + C(x^2 + 1)(x - 1) - (x^2 - 1)$

$= Ax^4 + (B + C)x^3 + (B - C - 1)x^2 + (B + C)x + (-A + B - C + 1)$

$x^4 \rightarrow A = 1$

$x^3 \rightarrow B + C = 0$

$x^2 \rightarrow B - C - 1 = 0$ (20)

$x^1 \rightarrow B + C = 0$

$x^0 \rightarrow -A + B - C + 1 = 1$

$\therefore \int \frac{x^4+1}{x^4-1} dx = \int \frac{A(x^4-1)+B(x^2+1)(x+1)+C(x^2+1)(x-1)-(x^2-1)}{x^4-1} dx$

$= A\int dx + B\int \frac{(x^2+1)(x+1)}{x^4-1} dfxc \int \frac{(x^2+1)(x-1)}{x^4-1} dx - \int \frac{x^2-1}{x^4-1} dx$ (05)

$= Ax + B\int \frac{1}{x-1} dx + C\int \frac{1}{x+1} dx - \int \frac{1}{x^2+1} dx$ (05)

$= x + \frac{1}{2} ln|x - 1| - \frac{1}{2} ln|x + 1| - \tan^{-1} x + k$ (20)

$= x + ln\sqrt{\frac{x-1}{x+1}} - \tan^{-1} x + k$ (05)

55

(b)

(i) $\frac{d(x+Cos\,x.Sin^3x)}{dx} = 1 + Cos\,x.3Sin^2x\ \ Cos\,x + Sin^3x(-Sinx)$ (05)

$= 1 + 3\,Sin^2x(1 - Sin^2x) - Sin^4x$

$= 1 + 3\,Sin^2x - 4Sin^4x$ (05)

$I = \int_0^{\pi/2} x\,(1 + 3\,Sin^2x - 4Sin^4x)dx$

$= \int_0^{\pi/2} x\,.\frac{d}{dx}(x + Cosx\,Sin^3x).\,dx$ (05)

$= [x(x + Cos\,x\,Sin^3x)]_0^{\pi/2} - \int_0^{\pi/2}(x + Cosx\,Sin^3x)\,dx$ (05)

$= \left[\frac{\pi}{2}\left(\frac{\pi}{2} + 0\right) - 0\right] - \int_0^{\pi/2} x\,dx - \int_0^{\pi/2} Sin^3x.\,Cos\,x\,dx$ (05)

$= \frac{\pi^2}{4} - \left[\frac{x^2}{2}\right]_0^{\pi/2} - \left[\frac{Sin^4x}{4}\right]_0^{\pi/2} = \frac{\pi^2}{4} - \frac{\pi^2}{8} - \frac{1}{4} = \frac{1}{8}(\pi^2 - 2)$ (05)

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(ii)

$I = \int_0^{\pi/2}(x + 3x\ \ Sin^2x - 4x\,Sin^4x)\,dx$

$= \int_0^{\pi/2}\left[\left(\frac{\pi}{2} - x\right) + 3\left(\frac{\pi}{2} - x\right)\ sin^2\left(\frac{\pi}{2} - x\right) - 4\left(\frac{\pi}{2} - x\right)Sin^4\left(\frac{\pi}{2} - x\right)\right]dx$ (05)

$= \int_0^{\pi/2}\left[\frac{\pi}{2} - x + 3\ \left(\frac{\pi}{2} - x\right)\ Cos^2x - 4\left(\frac{\pi}{2} - x\right)Cos^4x\right]dx$ (05)

$= \frac{\pi}{2}\ \int_0^{\pi/2}(1 + 3\,Cos^2x - 4Cos^2x)dx - \int_0^{\pi/2}(x + 3x\,Cos^2x - 4xCos^4x)\,dx$

$= \frac{\pi^2}{4} - \left[\frac{x^2}{2}\right]_0^{\pi/2} - \left[\frac{Cos^4x}{4}\right]_0^{\pi/2}$ (05)

$= \frac{\pi^2}{4} - \frac{\pi^2}{8} - \left[0 - \frac{1}{4}\right] = \frac{\pi^2}{8} + \frac{1}{4} = \frac{1}{8}(\pi^2 + 2)$ (05)

$\therefore I = \frac{\pi}{2}\mathcal{J}_1 - \mathcal{J}_2$

$\frac{1}{8}(\pi^2 - 2) = \frac{\pi}{2}\,\mathcal{J}_1 - \frac{1}{8}(\pi^2 + 2)$ (05)

$\frac{1}{8}(\pi^2 - 2) + \frac{1}{8}\ (\pi^2 + 2) = \frac{\pi}{2}\mathcal{J}_1$

$\mathcal{J}_1 = \frac{\pi}{2}$ (05)

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(c)

$\sqrt{x^3+1} = t$

$x^3 + 1 = t^2$

$3x^2 dx = 2t\, dt$ (05)

$x^3 = (t^2 - 1)$

$\binom{x=0}{t=\sqrt{1}=1}, \left(\begin{matrix} x=2 \\ t=\sqrt{2^3+1} \\ =3 \end{matrix}\right)$ (05)

$\int_0^2 \frac{x^8}{\sqrt{x^3+1}} dx = \int_0^2 \frac{(x^3)^2 \;\; x^2 dx}{\sqrt{x^3+1}}$ (05)

$= \int_1^3 \frac{(t^2-1)^2 \left(\frac{2}{3}\right) t\, dt}{t}$ (05)

$= \frac{2}{3} \int_1^3 (t^4 - 2t^2 + 1) dt$ (05)

$= \frac{2}{3}\left[\frac{t^5}{5} - 2\frac{t^3}{3} + t\right]_1^3$ (05)

$= \frac{2}{3}\left[\left(\frac{243}{5} - 18 + 3\right) - \left(\frac{1}{5} - \frac{2}{3} + 1\right)\right]$

$= \frac{2}{3}\left[\frac{242}{5} + \frac{2}{3} - 16\right]$

$= \frac{2}{3}\left(\frac{726+10-240}{15}\right)$

$= \frac{2}{3}\left(\frac{496}{15}\right)$

$= \frac{992}{45}$ (05)

35

16 $l_1: x - \sqrt{3}y + 1 + k = 0$ and $l_2: x + \sqrt{3}y + 1 - k = 0$ are two given straight lines passing through the point $(-1, 3)$ show that $k = 3\sqrt{3}$.

For that value of k, find the equations of the angle bisectors between the straight lines $l_1 = 0$ and $l_2 = 0$.

Let, l be the acute angle bisector of l_1 and l_2. Show that the point $A \equiv (2,3)$ lies on the line $l = 0$.

Find the equation of the circle S with centre A and the length of the diameter is 3 units.

Find the perpendicular distance from the point A to the line $l_1 = 0$, hence find the equation of the tangent drawn from $(-1, 3)$ to the circle S.

From a point P on the line $l = 0$, two tangents are drawn to the circle S so that they are perpendicular to each other.

Show that there are two such points for P and in each case find the coordinates.

Further, find the area of the quadrilateral which enclosed by the tangents.

$l_1 = x - \sqrt{3}\,y + 1 + k = 0$

$(If\ parallel\ through\ (-1,3))$

$-1 - 3\sqrt{3} + 1 + k = 0$ (05)

$k = 3\sqrt{3}$ (05)

$l_2 = x + \sqrt{3}y + 1 - k = 0$

$-1 + 3\sqrt{3} + 1 - k = 0$ (05)

$k = 3\sqrt{3}$

$l_2 = x + \sqrt{3}y + 1 - 3\sqrt{3} = 0$

$\therefore l_1 = x - \sqrt{3}y + 1 + 3\sqrt{3} = 0$

$$\frac{|x - \sqrt{3}y + 1 + 3\sqrt{3}|}{2} = \frac{|x - \sqrt{3}y + 1 - 3\sqrt{3}|}{2}$$ (10)

$(+)\ x - \sqrt{3}y + 1 + 3\sqrt{3} = x + \sqrt{3}y + 1 - 3\sqrt{3}$

$6\sqrt{3} = 2\sqrt{3}y$

$y = 3$ (05)

$(+)\ x - \sqrt{3}y + 1 + 3\sqrt{3} = -x - \sqrt{3}y - 1 + 3\sqrt{3}$

$2x = -2$

$x = -1$ (05)

$Consider\ \ y = 3\ then\ m = 0$ (05)

$l_1 = x - \sqrt{3}y + 1 + 3\sqrt{3} = 0 \qquad m_1 = \frac{1}{\sqrt{3}}$ (05)

$\tan\alpha = \left|\frac{\frac{1}{\sqrt{3}} - 0}{1 + 0}\right| = \frac{1}{\sqrt{3}} < 1\ (\therefore\ y = 3\ is\ \ an\ acute\ angle\ bisector)$ (05)

(10) (05)

$A \equiv (2,3) lies\ an\ the\ line\ y = 3$ (05)

70

$S \equiv (x-2)^2 + (y-3)^2 = \left(\frac{3}{2}\right)^2$ (10)

$x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{9}{4}$

$4x^2 + 4y^2 - 16x - 24y + 43 = 0$ (05)

15

The perpendicular distance from A (2,3) to the line $(l_1 = 0)$ = $\frac{|2-3\sqrt{3}+1+3\sqrt{3}|}{2} = \frac{3}{2}$ (05)

(10)

$\therefore$ *equation of the tangents* $l_1 = 0, l_2 = 0$ (10)

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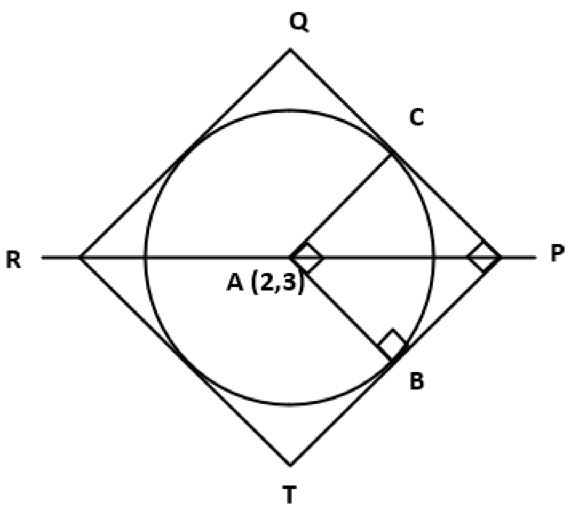

The tangent are perpendicular to each other. Then ABPC is a square. (05)

$AB = AC = \frac{3}{2}$ $\therefore AP = \frac{3\sqrt{2}}{2}$ ($\because$ *P lies on* $y = 3$) (05)

(05)

$\therefore$ *the position of P are* $\left(2 + \frac{3\sqrt{2}}{2}, 3\right)$ *and* $\left(2 - \frac{3\sqrt{2}}{2}, 3\right)$

(05) (05)

since, tangents are perpendicular, PQRT is a square (05)

$\therefore$ *Area of ABPC* $= 3 \times 3 = 9$ *Square units.*

(05) (05)

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17 (a) (i) Write down $\cos(A+B)$ In terms of $\cos A$, $\cos B$, $\sin A$, $\sin B$ and obtain an expression for $\cos 3A$ in terms of $\cos A$.

(ii) Determine constants λ and k such that,

$$\frac{2\cos 3x - 4cos^5x + 3cos^3x}{cosx(1+sin^2x)} = \lambda\cos 2x + k$$

Hence, find the maximum and minimum values of

$$f(x) = \frac{2\cos 3x - 4cos^5x + 3cos^3x}{cosx(1+sin^2x)}$$

and sketch the graph of $y = f(x)$ for $x\epsilon[-\pi, \pi]$

(b) A point P is inside the triangle ABC, such that $P\hat{A}B = P\hat{B}C = P\hat{C}A = \alpha$

By applying **Sine Rule** for suitable triangles, write down two expressions for PC_1 and show that $cot\alpha = cotA + cotB + cotC$

(c) Solve the equation $2tan^{-1}(cosx) = tan^{-1}(2cosecx)$ for $x\epsilon\left(0, \frac{\pi}{2}\right)$.

(a)(i) $cos(A+B) = cosAcosB - sinAsinB$

Substituting $B = 2A$ (05)

$cos3A = \cos(A+2A)$

$= cosAcos2A - sinAsin2A$ (05)

$= cosA(2cos^2A - 1) - sinA2sinAcosA$ (10)

$= 2cos^3A - cosA - 2cosA(1 - cos^2A)$ (05)

$= 2cos^3A - cosA - 2cosA + 2cos^3A$

$= 4cos^3A - 3cosA$ (05)

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(ii) $\frac{2cos3x-4cos^5x+3cos^3x}{cosx(1+sin^2x)} = \frac{2(4cos^3x-3cosx)-4cos^5x+3cos^3x}{cosx(1+sin^2x)}$ (05)

$= \frac{cosx(11cos^2x-4cos^4x-6)}{cosx(1+sin^2x)}$ (05)

$= \frac{11cos^2x - 4cos^4x - 6)}{1+sin^2x}$

$= \frac{(3-4cos^2x)(cos^2x-2)}{1+sin^2x}$ (05)

$= \frac{\left[3-4\left(\frac{1+cos2x}{2}\right)\right]\left[1-sin^2x-2\right]}{1+sin^2x}$ (05)

$= \frac{-(1-2cos2x)(1+sin^2x)}{1+sin^2x}$

$= 2cos2x - 1$

$= \lambda cos2x + k$; where $\lambda = 2,\ k = -1$ (05)

$f(x) = \frac{2cos3x - 4cos^5x + 3cos^3x}{cosx(1+sin^2x} = 2cos2x - 1$

$-1 \leq cos2x \leq 1$

$-2 \leq 2cos2x \leq 2$

$-3 \leq 2cos2x - 1 \leq 1$

Maximum value = 1   minimum value = -3

(05)   (05)

For maximum value

$2cos2x - 1 = 1$

$cos2x = 1$

$cos2x = cos0$

$2x = 2n\pi \; ; n \in \mathbb{Z}$

$x = n\pi$

$when\; n = -1, \quad x = -\pi$

$when\; n = 0, \quad x = 0$

$when\; n = \pi, \quad x = \pi$

$(-\pi, 1), (\pi, 0), (\pi, 1)$

For minimum value

$2cos2x - 1 = -3$

$cos2x = -1$

$cos2x = cos\pi$

$2x = 2n\pi \pm \pi \;\; ; n \in \mathbb{Z}$

$x = n\pi \pm \frac{\pi}{2}$

$when\; n = 0,$

$x = \pm\frac{\pi}{2}$

$\left(-\frac{\pi}{2}, -3\right), \left(\frac{\pi}{2}, -3\right)$

$when\; x = 0, \quad y = 1$

$when\; y = 0, \quad 2cos2x - 1 = 0$

$cos2x = \frac{1}{2}$

$cos2x = cos\frac{\pi}{3}$

$2x = 2n\pi \pm \frac{\pi}{3}$

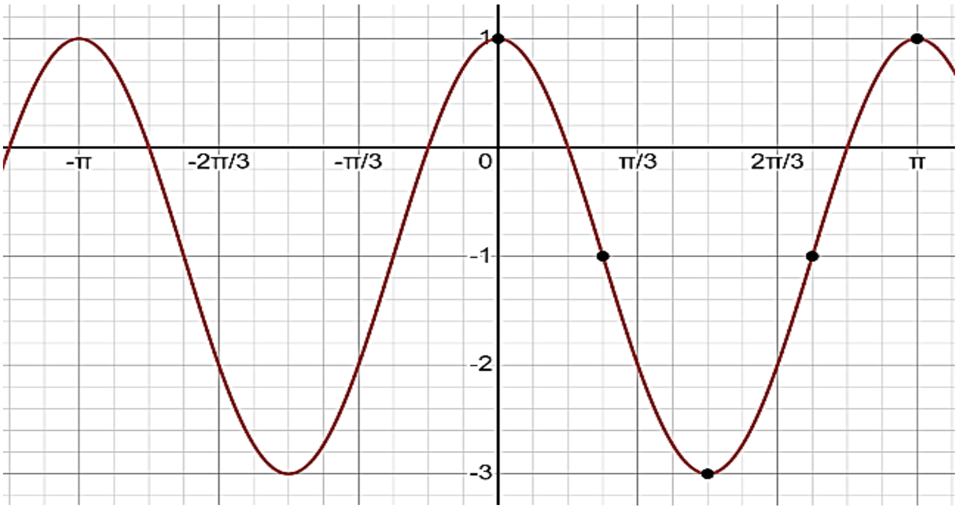
$x = n\pi \pm \frac{\pi}{6}$

$when\; n = 0, \quad x = \frac{\pm\pi}{6}$

$when\; n = 1, \quad x = \pi \pm \frac{\pi}{6}$

$when\; n = -1, \quad x = -\pi \pm \frac{\pi}{6}$

$\left(\pm\frac{\pi}{6}, 0\right), \left(\frac{5\pi}{6}, 0\right), \left(-\frac{5\pi}{6}, 0\right)$



For maximum (05)

For minimum (05)

Point of intersection (05)

End points (05)

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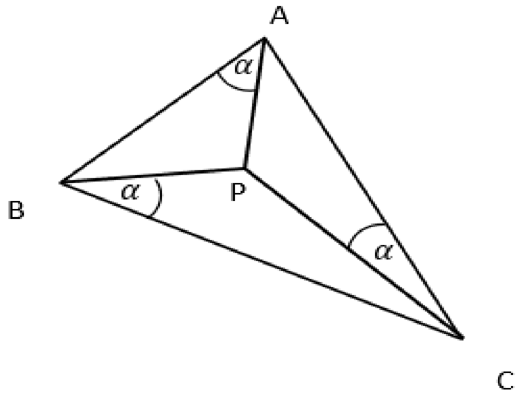
(b)



Applying Sin rule for triangle PBC

$$\frac{PC}{sin\alpha} = \frac{a}{\sin(180-c)} \Rightarrow PC = \frac{asin\alpha}{sinc}$$ (05) + (05)

Applying Sin rule for triangle PAC

$$\frac{PC}{\sin(A-\alpha)} = \frac{b}{\sin(180-A)} \Rightarrow PC = \frac{bsin(A-\alpha)}{sinA}$$ (05) + (05)

$$\frac{asin\alpha}{sinc} = \frac{bsin(A-\alpha)}{sinA}$$ (05)

$$ksin^2Asin\alpha = ksinBsinCsin(A-\alpha) \quad ; k \neq 0$$

$$sinAsin(B+C)sin\alpha = sinBsinCsin(A-\alpha)$$ (05)

$$sinAsin\alpha(sinBcosC + cosBsinC) = sinBsinC(sinAcos\alpha - cosAsin\alpha)$$ (05)

$$\frac{sinAsin\alpha(sinBcosC + cosBsinC)}{sinAsinBsinCsin\alpha} = \frac{sinBsinC(sinAcos\alpha - cosAsin\alpha)}{sinAsinBsinCsin\alpha}$$ (05)

$$cotC + cotB = cot\alpha - cotA$$ (05)

$$cot\alpha = cotA + cotB + cotC$$

45

(c)

$2tan^{-1}(cosx) = tan^{-1}2(cosecx)$

Let, take $tan^{-1}(cosx) = \alpha$

And$tan^{-1}2(cosecx) = \beta$

$tan\alpha = cosx$

$2\alpha = \beta$

$tan2\alpha = tan\beta$

$\frac{2tan\alpha}{1+tan^2\alpha} = 2cosecx$ (05)

$\frac{2cosx}{1-cos^2x} = 2cosecx$ (05)

$\frac{cosx}{1-cos^2x} = cosecx$

$\frac{cosx}{sin^2x} = \frac{1}{sinx}$

$sinxcosx - sin^2x = 0$

$tanx = 1 \because sinx \neq 0$ (05)

$tanx = tan\frac{\pi}{4}$

$x = n\pi \pm \frac{\pi}{4}\ ; n \in \mathbb{Z}$ (05)

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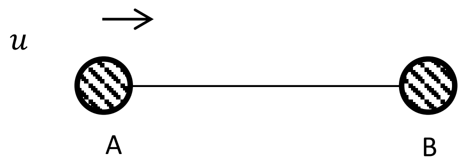
Ministry of Education

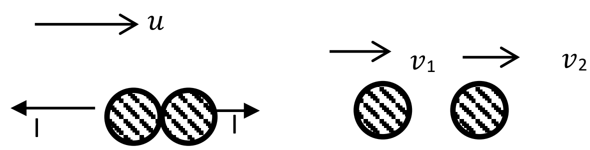
Support Seminar Paper-2023

10- Combined Mathematics II

Marking Scheme

01.





$\overrightarrow{I = \Delta MV}$ $\quad$ *for the system*

$0 = m(v_1 - u) + m(v_2 - 0)$ $\quad$ (05)

$v_1 + v_2 = u$ ——— (1)

$v_1 - v_2 = e(u - 0)$ $\quad$ (05)

$v_1 - v_2 = eu$ ——— (2)

(1) + (2) $\quad \Longrightarrow v_2 = \frac{u}{2}(1 + e)$

(1) - (2) $\quad \Longrightarrow v_1 = \frac{u}{2}(1 - e)$

$\rightarrow v_1 \rightarrow v_2 \quad \longrightarrow v_3 \longrightarrow v_3$

For system $A :\rightarrow I = \Delta mv$ $\quad$ (05)

$0 = m(v_3 - v_1) + m(v_3 - v_2)$

$v_3 = \frac{v_1 + v_2}{2}$

$v_3 = \frac{\frac{u}{2}.2}{2}$

$= \frac{u}{2}$

For $A \quad I = \Delta mv \rightarrow$ $\quad$ (05)

$I = m(v_3 - v_1)$

OR for partical B $\leftarrow$

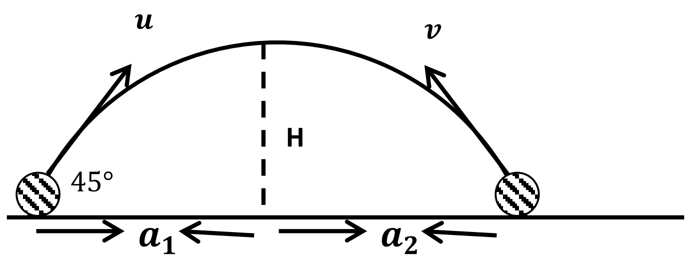
$I = m\big(-v_3 - (-v_2)\big)$

$= \frac{mue}{2}$ $\quad$ (05)

$= m\left(\frac{u}{2} - \frac{u}{2}(1-e)\right)$

$= \frac{mue}{2}$

Q2



For motion upto B

$A \uparrow \quad v = u + at$ for B $\uparrow v = u + at$ (05)

$0 = u\sin 45° - gt$ $\quad 0 = v \cos 60° - gt$

$t = \frac{u \sin 45°}{g}$ (05) $\quad t = \frac{v\cos 60°}{g}$

By equating t

$\frac{u\sin 45°}{g} = \frac{v\cos 60°}{g}$ (05)

$\frac{u}{\sqrt{2}} = \frac{\sqrt{3}v}{2}$

$\frac{u}{v} = \frac{\sqrt{3}v}{\sqrt{2}}$

$\therefore \quad u : v = \sqrt{3} : \sqrt{2}$

$a_1 + a_2 = a$ (05)

$\frac{u \cos 45°.u\sin 45°}{g} + \frac{v \cos 60°.v\sin 60°}{g} = a$

$\frac{u^2}{2g} + \frac{\sqrt{3}v^2}{4g} = a$

$2u^2 + \sqrt{3}v^2 = 4ag$

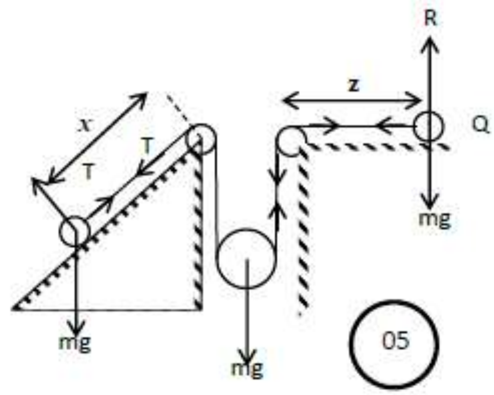
$2u^2 + \sqrt{3}\,\frac{2}{3}\,u^2 = 4ag$

$6u^2 + 2\sqrt{3}\,u^2 = 12ag$

$u^2 = \frac{6\,ag}{(3+\sqrt{3})} = \frac{2\sqrt{3}\ ag}{\sqrt{3}+1}$ 05

$u = \sqrt{\frac{2\sqrt{3}\ ag}{\sqrt{3}+1}}$

3)



$x + 2y + z = l$

$\ddot{x} + 2\ddot{y} + \ddot{z} = 0$

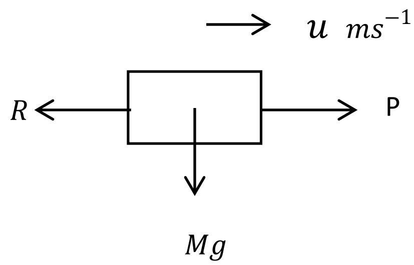
$\ddot{y} = -\frac{(\ddot{x}+\ddot{z})}{2}$ ——— (1) (05)

For $P \swarrow$ $mg \sin\alpha - T = m\ddot{x}$ ——— (2) (05)

For $Q \leftarrow T = -m\ddot{z}$ ——— (3) (05)

For $R \downarrow$ $M - 2T = M\ddot{y}$ ——— (4) (05)

04.



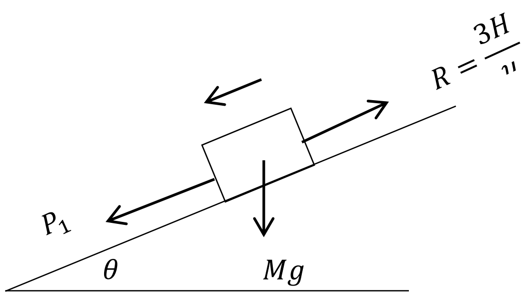
$P - R = M \times O$

$P = R$

$Pu = 3H \Rightarrow Ru = 3H$

$R = 3H \Rightarrow Ru = 3H$

$$R = \frac{3H}{u} N$$ (5)



$\sin\theta = \frac{1}{30}$

↙ $P_1 - \mathrm{R} + \mathrm{Mg}\sin\theta = M \times O$ (10)

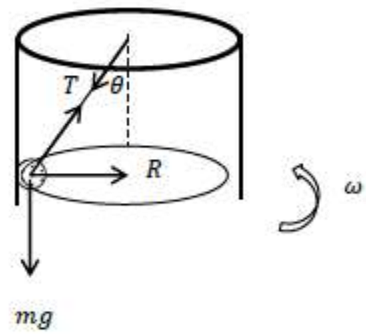
$P_1 = \mathrm{R} - \mathrm{Mg} \times \frac{1}{30} = \frac{3H}{u} - \frac{Mg}{30}$ (5)

$P_1 \mathrm{V} = 3\mathrm{H}$ (5)

$\left(\frac{3H}{u} - \frac{Mg}{30}\right)\mathrm{V} = 3\mathrm{H}$

$$V = \frac{3H \times 30u}{90H - Mgu} \; ms^{-1}$$

(5)



(05)

m

$\uparrow F = ma$

$\uparrow T\cos\theta = mg$

$T = \frac{2mg}{\sqrt{3}}$ (05)

$Sin\theta = \frac{a}{2a} = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ (05)

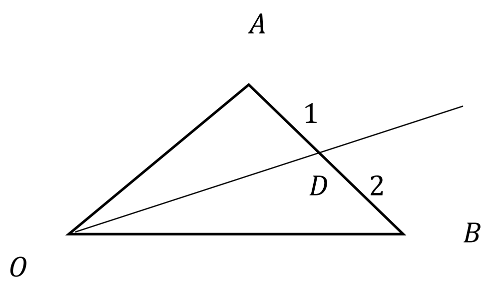
$\rightarrow F = ma$

$R + T\sin\theta = m.a\omega^2$ (05)

$R = ma\omega^2 - \frac{2mg}{\sqrt{3}} \cdot \frac{1}{2}$

$R = m\frac{(\sqrt{3}a\omega^2 - g)}{\sqrt{3}}$ (05)

06.



$\overrightarrow{OD} = \frac{2\times\overrightarrow{OA} + 1\times\overrightarrow{OB}}{2+1}$ (5)

$\overrightarrow{OD} = \frac{2(\underline{i}+\underline{j}) + 1(4\underline{i}+\underline{j})}{3}$

$\overrightarrow{OD} = \frac{6\underline{i}+3\underline{j}}{3}$

$\overrightarrow{OD} = 2\underline{i} + \underline{j}$ (5)

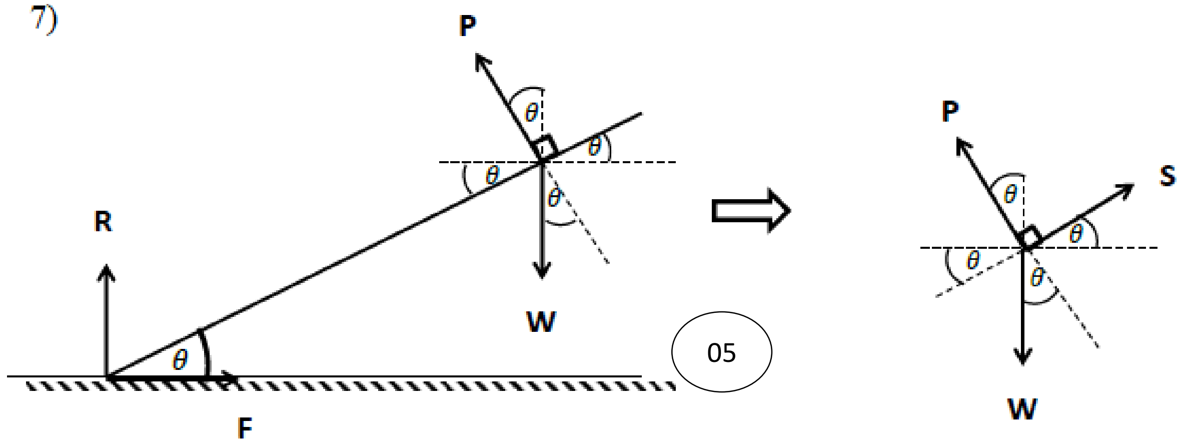
$\overrightarrow{OC} = 6\underline{i} + 3\underline{j} = 3(2\underline{i} + \underline{j})$ (5)

$\overrightarrow{OC} = 3\overrightarrow{OD}$ (5)

$\therefore OC\ //\ OD (\because point\ O\ is\ common)$

$\therefore O, C\ and$ D Collinear. (5)

7)



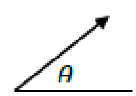
$$\frac{S}{\sin(\pi-\theta)}=\frac{S}{\sin\left(\frac{\pi}{2}+\theta\right)}=\frac{S}{\sin\left(\frac{\pi}{2}\right)}$$ (10)

$$\frac{S}{\sin\theta}=\frac{P}{\cos\theta}=W$$ (05)

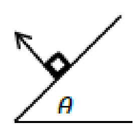
$S = W\sin\theta$

$P = W\cos\theta$

By considering the equibbrium

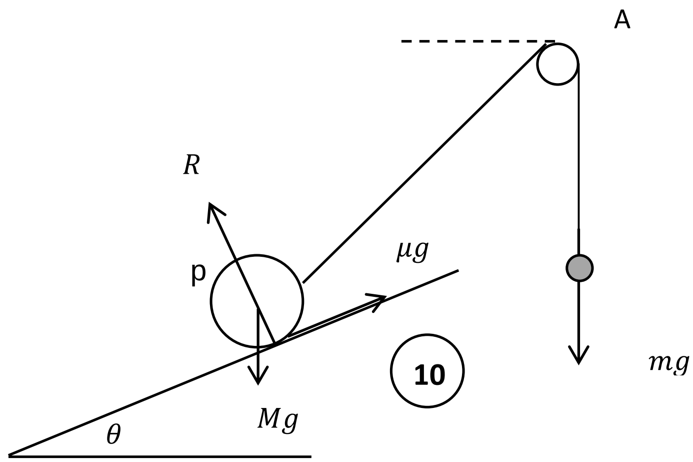


$S = W\sin\theta$ (05)



$P = W\cos\theta$ (05)

8.



P අංශුවට $\longrightarrow$ $\mu R\cos\theta + T\ \sin\theta - mg\sin\theta = 0$

5

$Q \downarrow mg - T = 0$ 5

$T = mg$

$P\ \uparrow R\cos\theta + T\cos\theta + \mu R\sin\theta - Mg = 0$ 5

Q (9) $\because$ $P(A/B) = P(B/C) = 0$

$P\ (A \cap B) = \emptyset$ *and* $P\ (B \cap C) = \emptyset$

$\because A, B$ *mutually exclusive and* B, C *mutually exclusive.*

$A \cap B \cap C = (A \cap C) \cap B = \emptyset$ 05

$P(A \cap B \cap C) = 0$

$\because\ P\ (A/C) = P\ (A) = P\ (A \cap C) = P\ (A).P(C) = 3k^2$ 05 05

$P\ (A \cup B \cup C) = P(A) + P\ (B) + P\ (C) - P\ (A \cap B) - P\ (B \cap C) - P\ (A \cap C) + P\ (A \cap B \cap C)$

$= 3k + 2k + k - 0 - 0 - 3k^2 + 0$

$\frac{11}{12} = 6k - 3k^2$ 05

$\therefore\ 36k^2 - 72k + 11 = 0$

$(6k - 1)\ (6k - 11) = 0$

$\because\ 6k - 11 \neq 0,\ \ k = \frac{1}{6}$ 05

Q (10)

x	-2	-1	0	1	2
f	4	1	3	1	1
fx	-8	-1	0	1	2
fx^2	16	1	0	1	4

05

$\overline{x} = \frac{\sum fx}{\sum f} = -\frac{6}{10} = -0.6$

05

$\sigma^2 x = \frac{\sum fx^2}{\sum f} - \overline{x}^2 = \frac{22}{10} - 0.36 = 1.84$

05

$Let\ y = 2000 - 4x$

$Then\ \overline{y} = 2000 - 4\overline{x} = 2000 + 2.4$

05

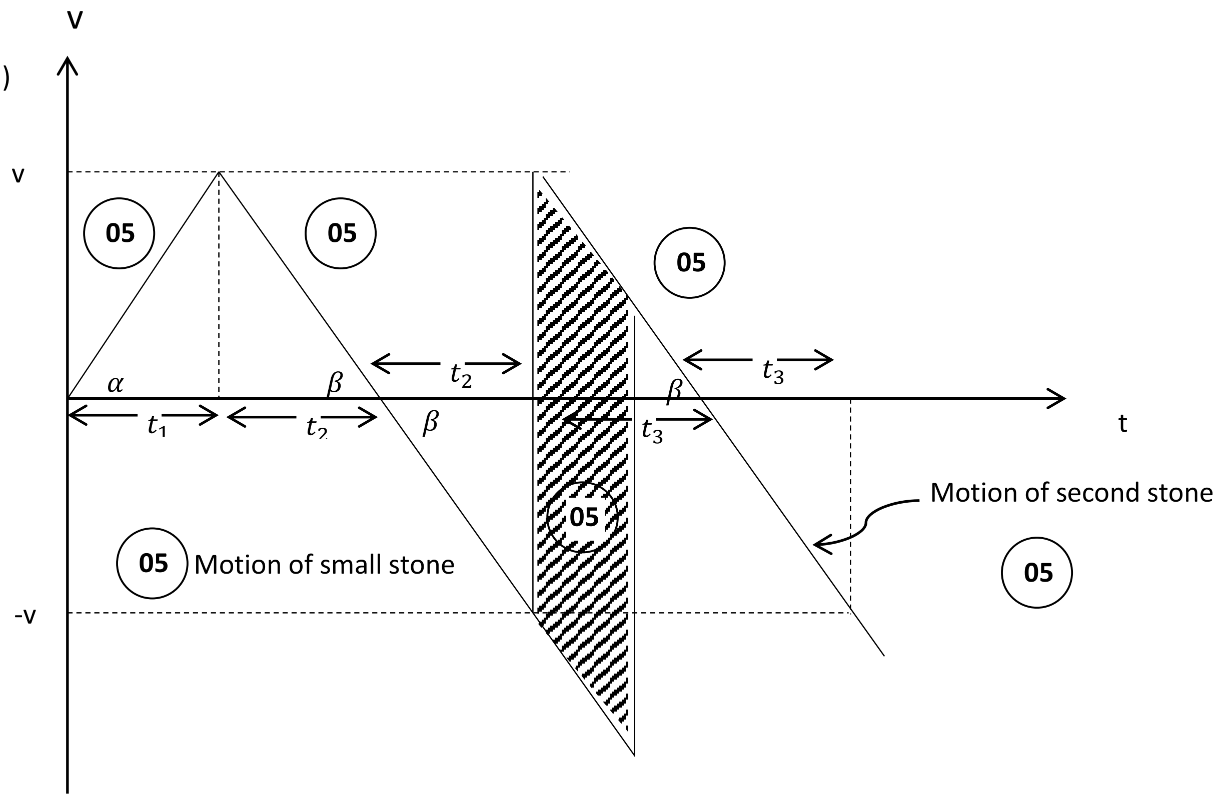
$= 2002.4$

$\sigma^2 y = 4^2 \sigma^2 x = 16 \times 1.84$

$\sigma y = \nless \sqrt{1.84}$

05

Q.11(a)



(ii) For the motion of small stone

$tan\alpha = \lambda g = \frac{v}{t_1} \Rightarrow v = \lambda g t_1$ 05

$H = \frac{1}{2} t_1 v$

$H = \frac{1}{2}\frac{v}{\lambda g} v \Rightarrow v = \sqrt{2\lambda g h}$ 05

(iii) Height attained by particle small

$H + \frac{1}{2} v t_2$ $where\ g = \frac{v}{t_2} \Rightarrow t_2 = \frac{v}{g}$ 05

$H + \frac{1}{2}.v.\frac{v}{g}$ 05

$H + \frac{v^2}{2g}$

(iv) Time taken to collide is equal to t

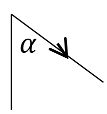
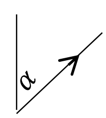
$\frac{1}{2}(2v + 2v).t = h$ **(05)**

$t = \frac{h}{2v}$

$= \frac{h}{2\sqrt{2\lambda gh}}$ **(05)**

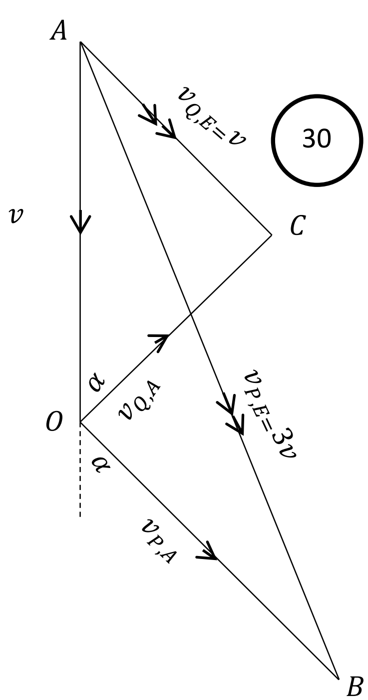
$= \sqrt{\frac{h}{8\lambda g}}$

Question 11 (b)

$v_{A,\ E} = \downarrow v$ $\quad v_{P\ \ A} =$  $\quad v_{Q\ \ A} =$ 

$v = 3v \qquad v_{Q,\ E} = v$ --------------(5)

By Relative velocity principle



$v_{P,E} = v_{P,A} + v_{A,E}$
$3v = \searrow + \ v \downarrow$ ----------------(5)
$v_{Q,E} = v_{Q,A} + v_{A,E}$
$v = \nearrow\ + \downarrow v$ ----------------(5)

$OAB\ for\ the\ motions\ of\ A\ and\ P$

$OAC\ for\ the\ motions\ of\ A\ and\ Q$

Let $v_{P,A} = v_1$
$v_{Q,A} = v_2$

$(v + v_1 cos\alpha)^2 + (v_1 sin\alpha)^2 = (3v)^2$
${v_1}^2 + 2vv_1 cos\alpha + v^2 = 9v^2$ ---------------(5)
${v_1}^2 + 2vv_1 cos\alpha + v^2cos^2\alpha + v^2sin^2\alpha = 9v^2$
$(v_1 + vcos\alpha)^2 = 9v^2 - v^2sin^2\alpha$

$v_1 + vcos\alpha = v\sqrt{9 - sin^2\alpha}$ ---------------(5)

$v_1 = v(\sqrt{9 - sin^2\alpha} - cos\alpha)$

$v_{P,A} = v(\sqrt{9 - sin^2\alpha} - cos\alpha)$ --------------(5)

$(v - v_2cos\alpha)^2 + v_2sin^2\alpha = v^2$

$v^2 + v_2{}^2 - 2vv_2cos\alpha = v^2$

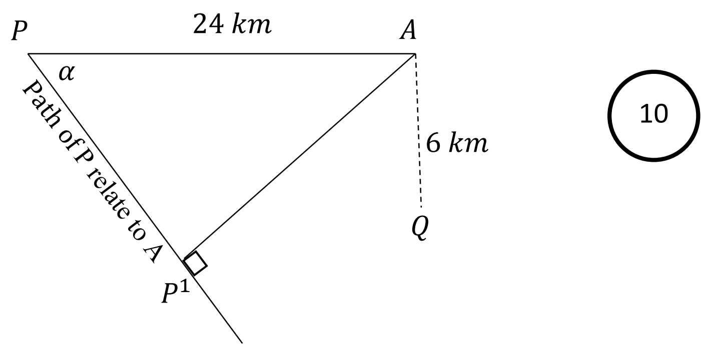
$(v_2 - vcos\alpha)^2 + v^2sin^2\alpha = v^2$

$v_2 = v\{\sqrt{1 - sin^2}\alpha + cos\alpha\}$

$v_{Q,A} = v\{\sqrt{1 - sin^2}\alpha + cos\alpha\}$ ---------(5)

$v_{Q,A} = 2vcos\alpha$

(ii)



$$cos\alpha = \frac{PP^1}{24}$$

$PP^1 = 24cos\alpha$ ------------(5)

$$\text{Time taken} = \frac{\text{Distance travelled by P relatedto A}}{v_{P,A}}$$

$$= \frac{24cos\alpha}{v\{\sqrt{9 - sin^2\alpha} - cos\alpha\}}$$ --------------------(5)

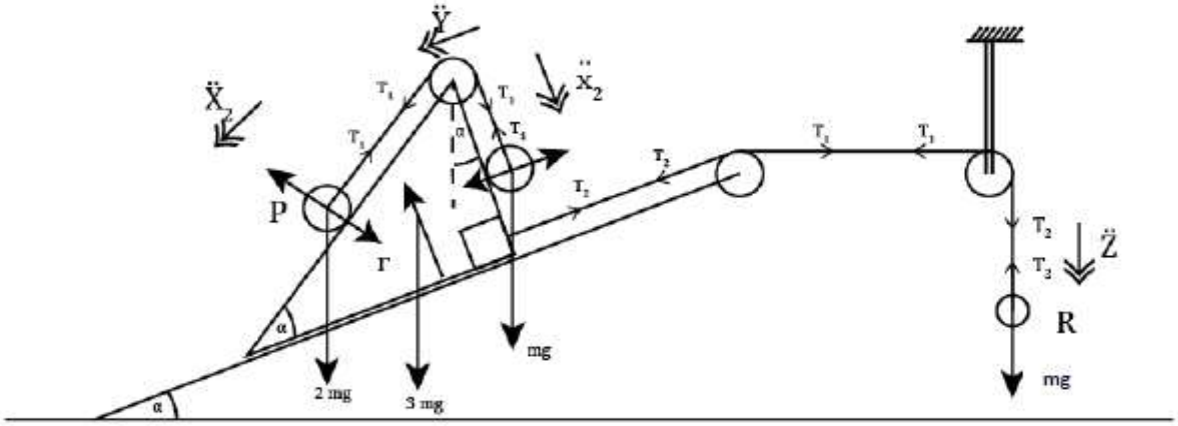
Distance travelled by Q at this time = $v_{P,A} \times t$

$$= 2vcos\alpha \times \frac{24cos\alpha}{v\{\sqrt{9 - sin^2\alpha} - cos\alpha\}}$$

$$= \frac{48cos^2\alpha}{\sqrt{\{\sqrt{9 - sin^2\alpha} - cos\alpha\}}}$$ ---------------------------(5)

---

(12) a



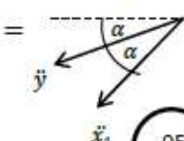
$x_1 + x_2 = l_1$

$\ddot{x}_1 + \ddot{x}_2 = 0$ ——— (1) (05)

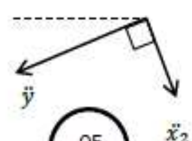
$y + z + k = l_2$

$\ddot{y} + \ddot{z} = 0$ ——— (2) (05)

$a_{YE} = \swarrow \ddot{y}$ $a_{RE} = \downarrow \ddot{z}$ $a_{PE} =$



$a_{QE} =$



(05) (05)

$F = ma$

For $P \swarrow 2mg \sin 2\propto - T_1 = 2m(\ddot{x}_1 + \ddot{y}\cos\propto)$ ——— (3) (10)

For $Q_1 \searrow mg\cos\propto - T_2 = m(\ddot{x}_2)$ ——— (4) (10)

For $R_1 \downarrow mg - T_2 = m(\ddot{z})$ ——— (5) (10)

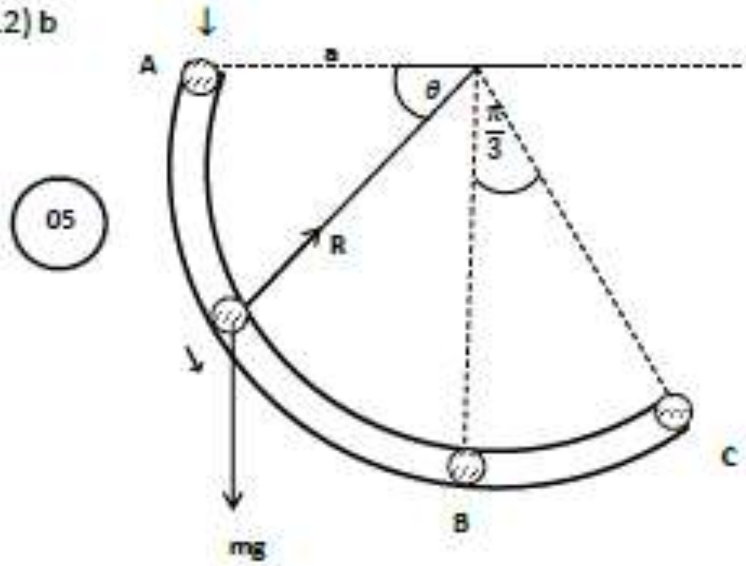
For the system P and Q

$T_2 - 6mg\sin\propto = 2m(-\ddot{y} - \ddot{x}_1\cos\propto) = 3m(\ddot{y}) + m(-\ddot{y})$ ——— (6) (20)

For $R \downarrow s = ut + \frac{1}{2}at^2$

$a = 0 + \frac{1}{2}\ddot{z}t^2$ ——— (7) (05)

(12) b

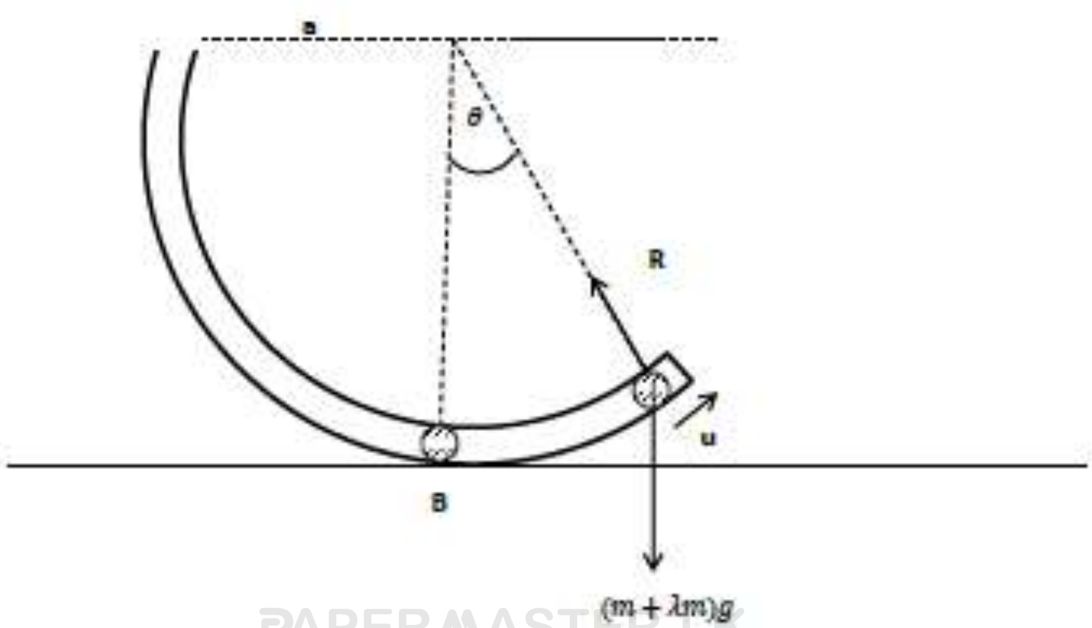


$0 = \frac{1}{2}mv^2 - mgasin\theta$ 05

$v^2 = 2gasin\theta$ 05

$v_1^2 = 2ga\,sin\frac{\pi}{2}$

$v_1 = \sqrt{2ga}$ 05



$\rightarrow I = \Delta mv$

$O = (m + \lambda m)v_2 - mv_1$ (05)

$v_2 = \frac{\sqrt{2ga}}{(1+\lambda)}$ (05)

$\frac{1}{2}(m + \lambda m)v_2^2 = \frac{1}{2}(m + \lambda m)u^2 + (m + \lambda m)g(a - acos\theta)$ (15)

$\frac{2ga}{2(1+\lambda)^2} = \frac{u^2}{2} + ga(1 - cos\theta)$

$u^2 = \frac{2ga}{(1+\lambda)^2} - 2ga(1 - cos\theta) = 2ga\left(\frac{1}{(1+\lambda)^2} + cos\theta - 1\right)$

$when\ u = 0\ ,\ \theta = \propto$

$\frac{1}{(1+\lambda)^2} + cos \propto -1 = 0$

$cos \propto = 1 - \frac{1}{(1+\lambda)^2}$ (05)

$when\ \propto < \frac{\pi}{3}$ (05)

$cos\alpha > cos\frac{\pi}{3}$

$1 - \frac{1}{(1+\lambda)^2} > \frac{1}{2}$

$\frac{1}{(1+\lambda)^2} < \frac{1}{2}$

$(1 + \lambda)^2 > 2 \rightarrow$

$1 + 2\lambda + \lambda^2 > 2$

$\lambda(\lambda + 2) > 1$ (05)

$\lambda = \sqrt{2} - 1$

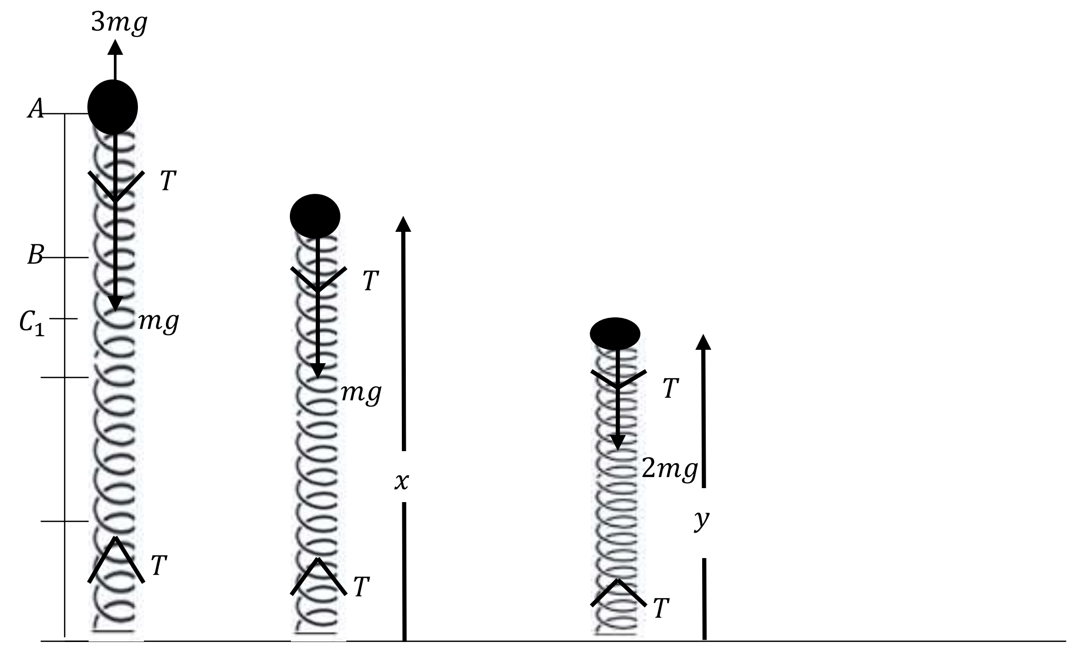
$\nwarrow F = ma$

$R - (m + \lambda m)gsin \propto = (m + \lambda m)\frac{v^2}{a}$ (05)

$R = m(1 + \lambda)g\ sin\frac{\pi}{3}$

$R = \sqrt{2}mg \times \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}}mg$ (05)

13.



When the particle at A

$\uparrow 3mg - mg - T = 0$

$2mg = \frac{\lambda l}{3l}$ (05)

$\lambda = 6mg$

15

When the partical lies between at B

$\uparrow\ -mg - T = m\ddot{x}$ (05)

$-mg - \frac{6mg(x-3l)}{3l} = m\ddot{x}$ (10)

$\ddot{x} = -\frac{2g}{l}\left(x - \frac{5l}{2}\right)$ (05)

$\ddot{x} = -\omega^2 X$ (05)

$At\ center\ X = 0 \ \leftrightarrow x = \frac{5l}{2}$

$\dot{x} = \dot{x} - \frac{5l}{2}$ (10)

$\ddot{X} = \ddot{x}$ (05)

30

$At\ A\ x = \frac{3l}{2}, \dot{x} = 0$

$\dot{x}^2 = \omega^2(C^2 - X^2)$ (05)

$0 = \omega^2\left(C^2 - \left(\frac{3l}{2}\right)^2\right)$ $\therefore Amplitude\ C = \pm\frac{3l}{2}$ (05)

$At\ B\ \dot{X}_B{}^2 = \frac{2g}{l}\left(\frac{9l^2}{4} - \left(\frac{l}{2}\right)^2\right)$ $\dot{X}_B = 2\sqrt{gl}$ (05)

15

$After\ Collision\ \downarrow I = \Delta mv$

$0 = m\left(+v - 2\sqrt{gl}\right) + m(v - 0)$ (05)

$V_B = \sqrt{gl}$ (05)

10

$When\ the\ partical\ lies\ between\ B\&D$

$\uparrow\ \underline{f} = m\underline{a}$

$T - 2mg = 2m\ddot{y}$ (05)

$\frac{6mg(3l-y)}{3l} - 2mg = 2m\ddot{y}$

(05)

$\ddot{y} = -\frac{g}{l}(y - 2l)$ (05)

15

$At\ the\ center\ \ddot{y} = 0 \qquad y = 2l$

$At\ B; t = 0, y = 3l\ , \dot{y} = \sqrt{gl}$

$y = 2l + \alpha\cos\omega t + \beta\sin\omega t$

(05) (05)

$3l = 2l + \alpha \quad \leftrightarrow \quad \alpha = l$

$\dot{y} = -\alpha\omega \sin \omega t + \beta\omega \cos \omega t$ (05)

$-\sqrt{gl} = \beta\omega$ (05)

$\ddot{y} = -\alpha\omega^2 \cos \omega t - \omega^2 sin\omega t$ (05)

$= -\omega^2(y - 2l)$ (05)

$\therefore \omega = \sqrt{\frac{g}{l}}$ $\qquad \beta = -l$

(05) (05)

40

$At\ the\ end\ of\ the\ amplitude\ \dot{y} = 0$

$\tan \omega t = -1$ (05) $\leftrightarrow$ $\omega t = \frac{3\pi}{4}$ (05)

$y = 2l + lcos\frac{3\pi}{4} - lsin\frac{3\pi}{4}$ (05)

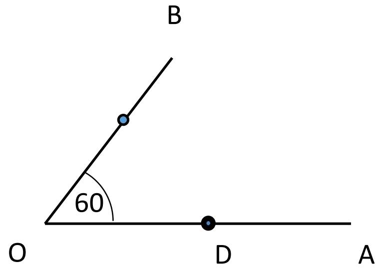
$= 2l - \frac{l}{\sqrt{2}} - \frac{l}{\sqrt{2}}$

$= 2l - \sqrt{2}l \quad \therefore amplitude = \sqrt{2}l$

(05) (05)

25

14. 14.I



A$(\sqrt{3}\underline{i}+\underline{j})$

$\overrightarrow{OA}.\overrightarrow{OB}=\left(\sqrt{3}\underline{i}+\underline{j}\right).(\alpha\underline{i}+\beta\underline{j})$ (05)

(05) $\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|\cos 60^\circ=\sqrt{3}\alpha+\beta$ (05)

$\sqrt{3+1^2}\times 10\times\frac{1}{2}=\sqrt{3}+\beta$

$10=\sqrt{3}\alpha+\beta$ (05)

$\overrightarrow{OB}=5\sqrt{3}\underline{i}-5\underline{j}$

$\alpha^2+\beta^2=10^2$ (05)

$\alpha^2+\left(10-\sqrt{3}\alpha\right)^2=100$

$\alpha^2+100+3\alpha^2-20\sqrt{3}\alpha=100$

$4\alpha^2-20\sqrt{3}\alpha=0$

$\alpha\left(\alpha-5\sqrt{3}\right)=0$

$\because \alpha\neq 0$

$\alpha=5\sqrt{3}$ (05)

$\beta=-5$ (05)

35

$OC:CB=1:\lambda$

$\overrightarrow{OC}=\frac{1}{\lambda+1}\quad\overrightarrow{OB}=\frac{1}{\lambda+1}(5\sqrt{3}\underline{i}-5\underline{j})$

(05)

$\overrightarrow{AC}=\overrightarrow{AO}+\overrightarrow{OC}$ (05)

$\frac{\sqrt{3}\underline{i}}{2}-\frac{5\underline{j}}{2}=-\sqrt{3}\underline{i}-\underline{j}+\frac{1}{(\lambda+1)}\left(5\sqrt{3}\underline{i}-5\underline{j}\right)$ (10)

$\frac{\sqrt{3}\underline{i}}{2}-\frac{5\underline{j}}{2}=-\sqrt{3}\underline{i}+\frac{5\sqrt{3}}{\lambda+1}\underline{i}-\underline{j}-\frac{5}{\lambda+1}\underline{j}$

Comparing Coefficients of : $\underline{i},\underline{j}$

$\underline{i} \rightarrow \quad -\sqrt{3} + \frac{5\sqrt{3}}{\lambda + 1} = \frac{\sqrt{3}}{2}$

$-2 + \frac{10}{\lambda + 1} = 1$

$\frac{10}{\lambda + 1} = 3$

$10 = 3\lambda + 3$

$\lambda = \frac{7}{3}$ (05)

$\underline{j} \rightarrow \quad -\frac{5}{2} = -1 - \frac{5}{\lambda + 1}$

$-5 = -2 - \frac{10}{\lambda + 1}$

$\frac{10}{\lambda + 1} = 3$

$10 = 3\lambda + 3$

$\lambda = \frac{7}{3}$ (05)

$OC : CB = 1 : \frac{7}{3}$

$OC : CB = 3 : 7$

(05)

35

$\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$

$= -5\sqrt{3}\underline{i} + 5\underline{j} + \frac{1}{2}\overrightarrow{OA}$

$= -5\sqrt{3}\underline{i} + 5\underline{j} + \frac{1}{2}(\sqrt{3}\underline{i} + \underline{j})$

$= \frac{1}{2}(-10\sqrt{3}\underline{i} + 10\underline{j} + \sqrt{3}\underline{i} + \underline{j})$

$\overrightarrow{BD} = \frac{-9\sqrt{3}\underline{i}}{2} + \frac{11\underline{j}}{2}$ (5)

$\overrightarrow{AE} = \frac{10}{17}\overrightarrow{AC} = \frac{10}{17}(\overrightarrow{AO} + \overrightarrow{OC})$ 5

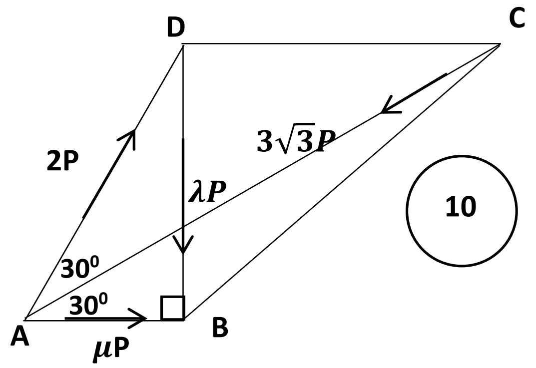
$\overrightarrow{AE} = \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \frac{3}{10}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)\right)$

$= \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \frac{3}{10}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)\right)$

$= \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \frac{3}{10}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)\right)$ 5

$= \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \left(\frac{3\sqrt{3}\underline{i}}{2} - \frac{3\underline{j}}{2}\right)\right)$

14.b



10

$B^{\curvearrowleft} = -3\sqrt{3}P \times AB\ Sin\ 30 + 2P \times AB\ Sin\ 60$ 10

$B^{\curvearrowleft} = -3\sqrt{3}P \times \frac{1}{2} \times AB + 2P \times \frac{\sqrt{3}}{2} \times AB = -\frac{\sqrt{3}\,P}{2} \times AB$

$B^{\curvearrowleft} = \frac{\sqrt{3}\,P}{2} \times AB \neq 0$ 5

∴ the sys from in not equilibrium Value of λ, μ

$A \curvearrowright = 0$

$\lambda P \times A\beta = 0$

$P \neq O \quad AB \neq 0 \quad \lambda = O$

10

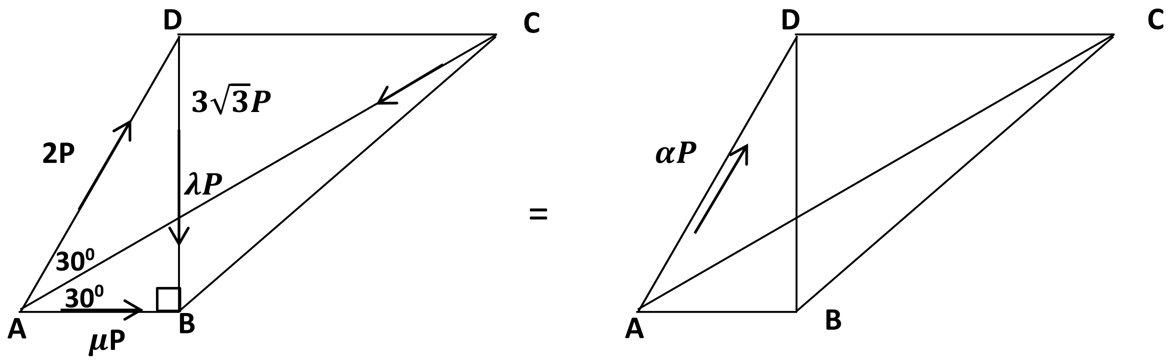
$D \curvearrowright = O$

- $\mu P \times DB \sin 60 + 3\sqrt{3}\ P \times AD \sin 30^0 = 0$

10

$\mu \times DB \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \times AD \times \frac{1}{2}$; $AD \neq 0$

$\underline{\underline{\mu = 3}}$

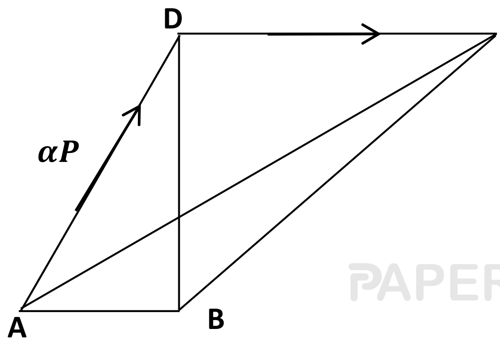


$C \curvearrowleft = 3P \times AC \sin 30 - 2P \times AC \sin 30 = \alpha P \times AC \sin 30$

10

$\frac{3}{2} - \frac{2}{2} = \frac{\alpha}{2}$

$\alpha = 1$ *if the line of action of new resultant* ……………………..

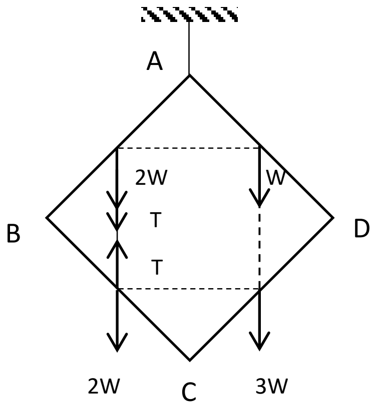


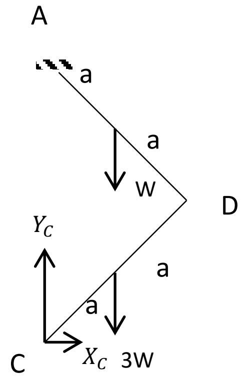
10

$G \circlearrowleft \quad D = 0$

$G = 0$

15 a)





$A \circlearrowleft$

$4W\,.\,a\cos\frac{\pi}{4} = X_C.\,4a\cos\frac{\pi}{4}$ **(10)**

$X_c = W$ **(05)**

DC **(10)**

$Y_C\,.2a\cos\frac{\pi}{4} = X_C.\,2a\cos\frac{\pi}{4} + 3W.\,a\cos\frac{\pi}{4}$

$\therefore Y_c = \frac{5}{2}W$ **(05)**

$\therefore\ C$

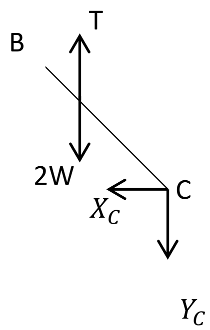
$R_C = \sqrt{{X_C}^2 + {Y_C}^2}$

$= \sqrt{W^2 + \frac{25}{4}W^2}$

$= \sqrt{W^2 + \frac{25}{4}W^2}$

$= \frac{\sqrt{29}}{2}W$ **(05)**

BC



10

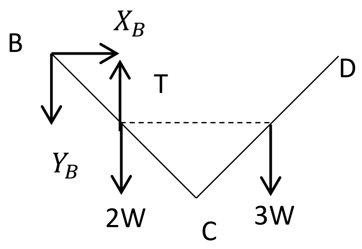
$B \curvearrowleft$

$$T.aCos\frac{\pi}{4} = 2W.aCos\frac{\pi}{4} + X_C.2acos\frac{\pi}{4} + Y_C.2acos\frac{\pi}{4}$$

$\therefore T = 2W + 2W + 5W$

$T = 9W$ **05**

BC+CD



$D \curvearrowleft$

$$Y_B.4aCos\frac{\pi}{4} + 2W.3aCos\frac{\pi}{4} + 3Wacos\frac{\pi}{4} = T.3acos\frac{\pi}{4}$$ **10**

$4Y_B + 6W + 3W = 3T = 3(9W)$

$\therefore Y_B = \frac{9}{2}W$ **05**

BC

$X_B = X_C = W$ **05**

$\therefore B$

$$R_B = \sqrt{W^2 + \frac{81}{4}W^2}$$

$= \frac{\sqrt{85}}{2}W$ **05**

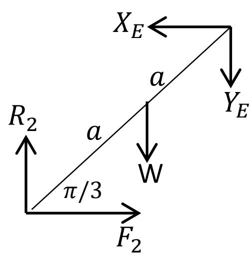
$A \quad R_2.2a + (3W + W)acos\frac{\pi}{3} = 2W.a + (W + W)\left(2a + acos\frac{\pi}{3}\right)$ --------------(10)

$R_2 = \frac{5}{2}W$ --------------(5)

↑ $R_1 + R_2 = 8W$

$\therefore R_1 = \frac{11}{2}W$ -------------(5)

Only the rod EF,



E $F_2 . 2a\cos\frac{\pi}{6} + W . a\cos\frac{\pi}{3} = R_2 . 2a\cos\frac{\pi}{3}$ --------------(10)

$F_2 = \frac{2}{\sqrt{3}} W$ ----------------(5)

↑ $R_2 = Y_E + W$

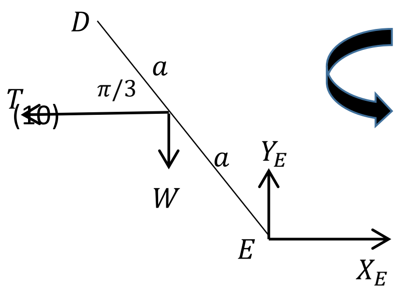
$Y_E = \frac{3}{2} W$ -------------------(5)

→ $X_E = F_2 = \frac{2}{\sqrt{3}} W$-------------------(5)

Entire system

→ $F_1 = F_2 = \frac{2}{\sqrt{3}} W$-------------------(5)

Only the rod DE



D $T . a\cos\frac{\pi}{6} + W . a\cos\frac{\pi}{3} = Y_E . 2a\cos\frac{\pi}{3} + X_E . 2a\cos\frac{\pi}{6}$ -----------(10)

$T = 2\sqrt{3}W$

For the equilibrium at A

$$F_1 \leq \mu R_1$$

$$\frac{2}{\sqrt{3}} W \leq \mu \frac{11}{2} W$$

$\frac{4}{11\sqrt{3}} \leq \mu$------------------(5)

For the equilibrium at F

$$F_2 \leq \mu R_2$$

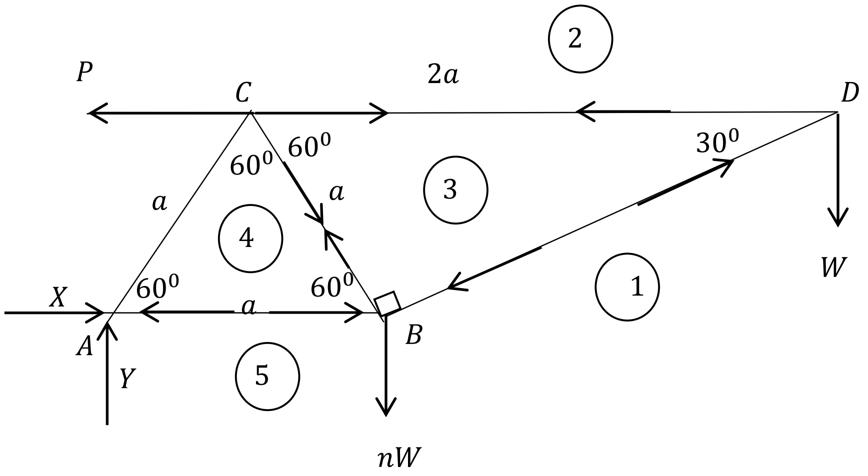
$$\frac{2}{\sqrt{3}}W \leq \mu \frac{5}{2}W$$

$\frac{4}{5\sqrt{3}} \leq \mu$------------------(5)

$\therefore If\ \frac{4}{11\sqrt{3}} < \mu < \frac{4}{5\sqrt{3}}$ then ------------------(5)

Even though the point A is in equilibrium, the point F is not in equilibrium.

b)



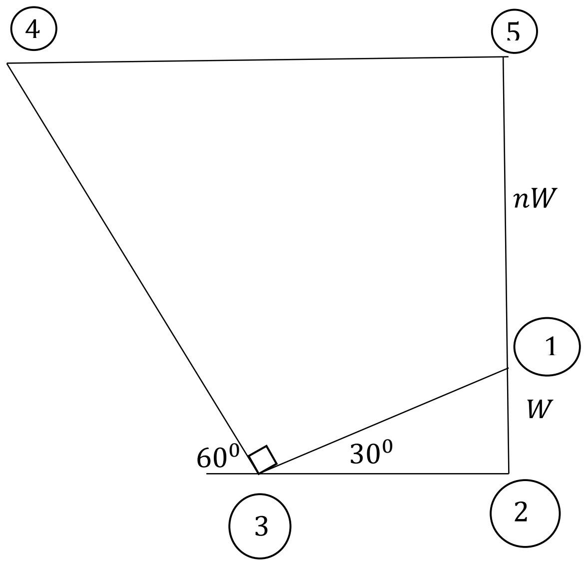
Lets take $AB = BC = AC = a$

$CD = 2a$

By considering the entire system,

A $P.a\cos\frac{\pi}{6} = nWa + w\left(2a + \frac{a}{2}\right)$ -------------------------(10)

$P = \left(\frac{2n+5}{\sqrt{3}}\right)W$



Rod	Tension	Thrust
AB	______________________	$\left(\frac{n+4}{\sqrt{3}}\right)W$-------(5+5)
BC	$\frac{2}{\sqrt{3}}$(n+1)W-------(5+5)	______________________
CD	$\sqrt{3}W$-------(5+5)	______________________
BD	______________________	$2W$

$(1)(3)cos\frac{\pi}{3} = (1)(2)$ $\quad$ $(2)(3) = (1)(3)cos\frac{\pi}{6}$ $\quad$ $(3)(4)cos\frac{\pi}{6} = nW + W$

$(1)(3) = 2W$ $\quad$ $= \sqrt{3}W$ $\quad$ $(3)(4) = \frac{2}{\sqrt{3}}(n+1)W$

$$(4)(5) = (2)(3) + (3)(4)cos\frac{\pi}{3}$$

$$(4)(5) = \sqrt{3}W + \frac{(n+1)}{\sqrt{3}}W$$

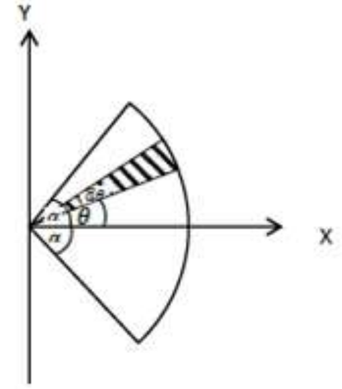
If the maximum possible tension for the rod BC is $10\sqrt{3}W$,

Then $\frac{2(n+1)}{\sqrt{3}}W \leq 10\sqrt{3}W$ -----------------(10)

$$n \leq 14$$

16.

(a) (1)



By the definition of center of mass.

$$\bar{x} = \frac{\int_{-\propto}^{+\propto} \frac{2}{3}r\cos\theta \, \frac{1}{2}r^2 d\theta \rho}{\int_{-\propto}^{\propto} \frac{1}{2}r^2 d\theta \, \rho}$$ (5) (5)

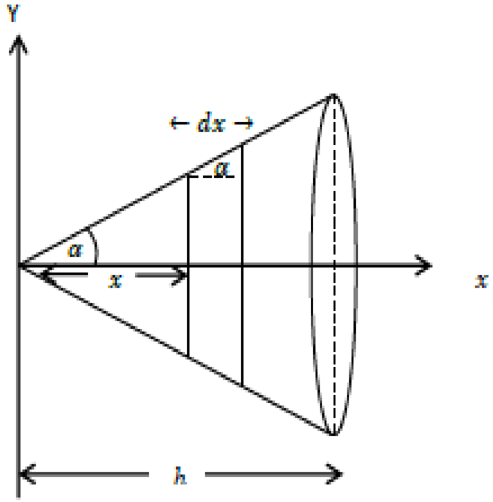
$$= \frac{\frac{1}{2}mr^2 \, \frac{2}{3}r\int_{-\propto}^{\propto}\cos\theta \, d\theta}{\frac{1}{2}mr^2 \int_{-\propto}^{\propto} d\theta} \qquad = \frac{2}{3}r\frac{[\sin\theta]_{-\propto}^{+\propto}}{[\theta]_{-\propto}^{+\propto}}$$ (5)

$$= \frac{2}{3}r\frac{[\sin\propto - \sin(-\propto)]}{[\propto-(-\propto)]} \qquad = \frac{2}{3}r\frac{2\sin\propto}{2\propto} \qquad = \frac{2r\sin\propto}{3\propto}$$ (5)

The center of mass of uniform secter lies on its symmetrical axis at a distance $\frac{2r\sin\propto}{3\propto}$ *from O.* (5)

25

(a) (II)



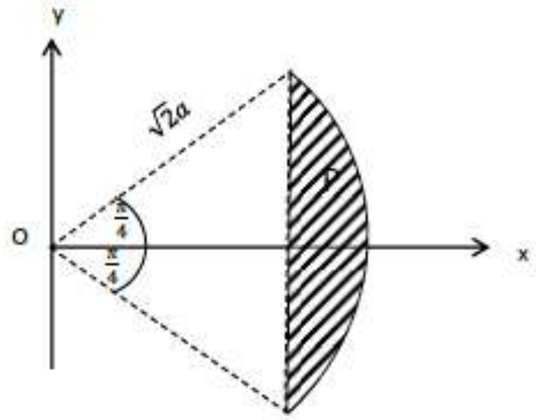
$$\bar{x} = \frac{\int_0^h 2\pi x \tan\alpha\, dx \sec\alpha\, \rho\, x}{\int_0^h 2\pi x \tan\alpha\, dx \sec\alpha\, \rho} \quad (5) \quad (5) \quad = \frac{2\pi \tan\alpha \sec\alpha\, \rho \int_0^h x^2}{2\pi \tan \sec \rho \int_0^h x}$$

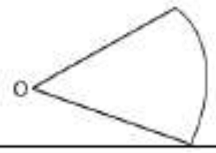
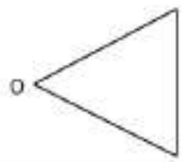
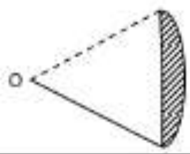
$$= \frac{\left[\frac{x^3}{3}\right]_0^h}{\left[\frac{x^2}{2}\right]_0^h} \quad (5)$$

$$= \frac{2}{3}h \quad (5)$$

The center of mass of uniform hollow cone lies on its symmetrical axis at a distance $\frac{2h}{3}$ *form O.* (5)

25



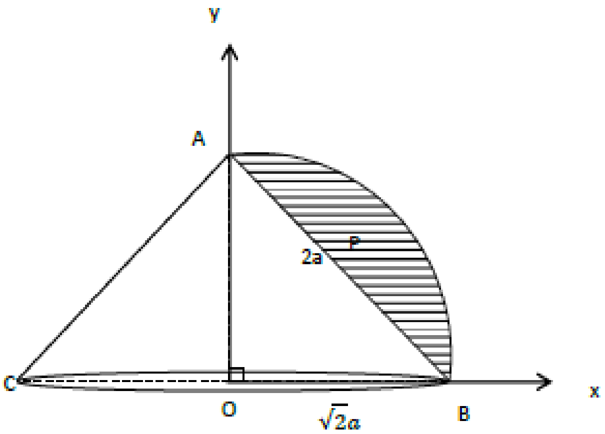
Object	Mass	$\bar{x}$
	$\frac{\pi a^2}{2}\rho$ (5)	$\frac{8a}{3\pi}$ (5)
	$a^2\rho$ (5)	$\frac{2}{3}a$ (5)
	$a^2(\pi/_2 - 1)\rho$ (5)	$\bar{x}$

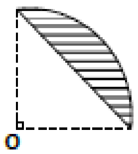
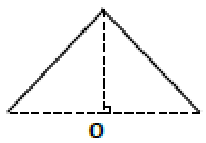
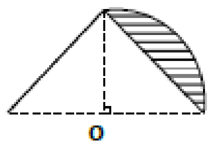
$$\bar{x} = \frac{\frac{\pi a^2}{2}\rho\frac{8a}{3\pi} - a^2\rho\frac{2}{3}a}{a^2(\pi/_2 - 1)} = \frac{\frac{4a}{3} - \frac{2a}{3}}{\pi/_2 - 1} = \frac{4a}{3(\pi - 2)}$$ (5)

(5)

Centre of gravity of the object is on the ox symmetric axis as it is symmetric about ox

40



Object	Mass	$\bar{x}$	$\bar{y}$
	M	$\frac{2\sqrt{2}a}{3(\pi-2)}$ (5)	$\frac{2\sqrt{2}a}{3(\pi-2)}$ (5)
	5M	$\frac{\sqrt{2}a}{3}$ (5)	0 (5)
	6M	$\bar{x}$	$\bar{y}$

$6M\bar{x} = \frac{M\times 2\sqrt{2}a}{3(\pi-2)} + 5M \times \frac{\sqrt{2}}{3}a$

$\bar{x} = \frac{2\sqrt{2}a + 5\sqrt{2}a(\pi-2)}{18(\pi-2)}$ (10)

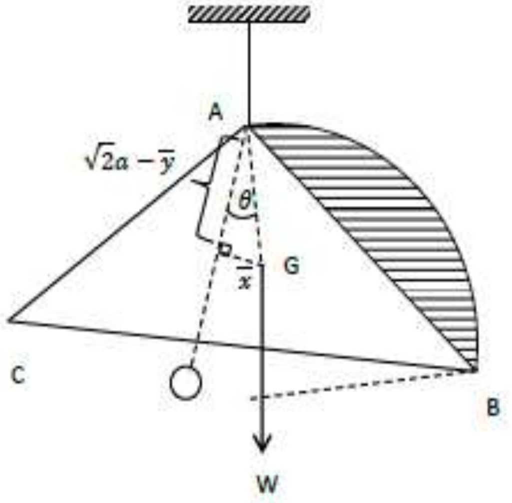
$= \frac{2+5(\pi-2)2\sqrt{2}a}{18(\pi-2)}$

$= \frac{(5\pi-2)\sqrt{2}a}{18(\pi-2)}$

$6M\bar{y} = \frac{M2\sqrt{2}a}{3(\pi-2)}$

$= \frac{\sqrt{2}a}{9(\pi-2)}$ (10)

40



(5)

$$\tan\theta = \frac{\bar{x}}{\sqrt{2}a - \bar{y}}$$ (5)

$$= \frac{\frac{a(5\pi-8)\sqrt{2}a}{18(\pi-2)}}{\sqrt{2}a - \frac{\sqrt{2}a}{9(\pi-2)}}$$ (5)

$$= \frac{(5\pi-8)\sqrt{2}a}{2[9\sqrt{2}a(\pi-2) - \sqrt{2}a]}$$

$$= \frac{5\pi-8}{2[9(\pi-2)-1]}$$ (5)

$$= \frac{5\pi-8}{2[9\pi-19]}$$

20

Part B (CM2)

17.

(a) $P(B) = 3, \quad P(B \cup C) = 0.37 \; and \quad P(C) = 0.2$

(i) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ (05)

$(\therefore A \; and \; B \; are \; independent)$

$0.37 = P(A) + o.3 - P(A).0.3$ (05)

$0.07 = P(A) \times 0.7 \Rightarrow P(A) = 0.1$

(ii) $P(B' \backslash A^1) = \frac{P(B' \cap A')}{P(A')}$ (05)

$P(B' \cap A') = P(B \cup A)' = 1 - P(B \cup A)$

$= 1 - 0.37 = 0.63$ (05)

$P(A') = 1 - P(A) = 1 - 0.1 = 0.9$

$\therefore \; P(B' \backslash A') = \frac{0.63}{0.9} = 0.7$ (05)

(iii) $P(A' \cap B' \cap C) = P(A')P(B')P(C)$ (05)

$= 0.9 \times 0.7 \times 0.2$

$= 0.126$ (05)

(iv) $Let \; X = \; (A \cap B' \cap C') \cup P(A' \cap B \cap C') \cup (A' \cap B' \cap C)$ (05)

$\therefore P(X) = P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= P(A) \; P(B') \; P(C') + P(A') \; P(B) \cup (C') + P(A')P(B')P(C)$ (05)

$= 0.1 \times 0.7 \times 0.8 + 0.9 \times 0.3 \times 0.8 + 0.9 \times 0.7 \times 0.2$

$= 0.398$ (05)

$\Rightarrow P(A/X) = \frac{P(A \cap X)}{P(X)}$

$= \frac{P(A \cap B' \cap C')}{P(X)}$ (05)

$= \frac{0.1 \times 0.7 \times 0.8}{0.398}$

$= \frac{28}{199}$ (05)

(b)

Distance	x_i	$y_i = \frac{x_i - 45}{10}$	f	fy	fy^2
0 – 10	05	-4	10	-40	160
10 – 20	15	-3	19	-57	171
20 – 30	25	-2	43	-86	172
30 – 40	35	-1	25	-25	25
40 – 50	45	0	8	0	0
50 – 60	55	1	6	6	6
60 – 70	65	2	5	10	20
70 – 80	75	3	3	9	27
80 – 90	85	4	1	4	16
			120	-179	597
	05	05		05	05

$\therefore\ y_i = \frac{x_i - 45}{10}$

$\therefore\ \bar{x} = 10\bar{y} + 45$ 05

$Henc\ \bar{y} = \frac{\sum fy}{\sum f} = \frac{-179}{120} = -1.49$ 05

$\therefore\ \bar{x} = 10(-1.49) + 45 = 30.08$ 05

$\sigma y^2 = \frac{\sum f y_c^2}{\sum f} - \bar{y}^2$ 05

$= \frac{1}{120}\ (597 - 120 \times 2.22)$ 05

$= \frac{1}{120}\ (597 - 266.40) = 2.76$ 05

05

$\sigma x^2 = 10^2 \sigma y^2 = 100 \times 2.76 = 276$ 05

$\therefore\ \sigma x = 16.61$ (05)

65

11. $Number\ transfixed\ =\ 15$

$\therefore$ *The new distribution has only* 1^{st} …………… ……………..*total number*

$120 - 15 = 105$

1^{st} …………………………….[10,20]

$\because Q_1 = \frac{1}{4} \times 105^{th}\ position =\ 26.25^{th}\ position$

$=\ 10\ +\ \frac{(26.25-10)}{19} \times 10$ (05)

$=\ 10\ + 8.55 = 18.55$ (05)

$3^{rd} Quater\ Q_3 = \frac{3}{4}(105)^{th}\ position$

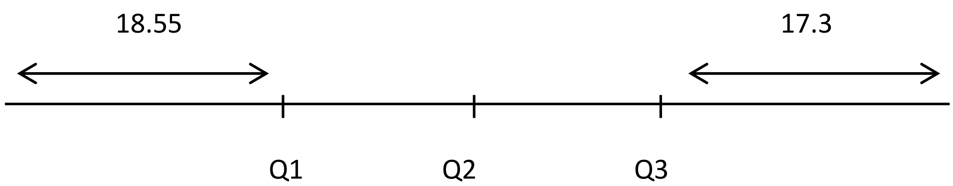
$= 78.75^{th}\ position$

The requared …………………..*is* [30,40]

$Q_3 = 30 + \frac{(76.75-72)}{25} \times 10$ (05)

$= 30 + 2.7 = 32.7$ (05)

$\therefore IQR = 32.7 - 18.55 = 14.15$ (05)



$\therefore$ *The distribution is approximately Symmetric.* (05)

30