அந்தரங்கமானது

இலங்கைப் பரீட்சைத் திணைக்களம்

க.பொ.த (உயர் தர)ப் பரீட்சை – 2022(2023)

10 - இணைந்த கணிதம் I

புள்ளியிடும் திட்டம்

இந்த விடைத்தாள் பரீட்சகர்களின் உபயோகத்திற்காகத் தயாரிக்கப்பட்டது. பிரதம பரீட்சகர்களின் கலந்துரையாடல் நடைபெறும் சந்தர்ப்பத்தில் பரிமாறிக்கொள்ளப்படும் கருத்துக்களுக்கேற்ப இதில் உள்ள சில விடயங்கள் மாற்றப்படலாம்.

இறுதித்திருத்தங்கள் உள்ளடக்கப்படவுள்ளன

க.பொ.த (உயர் தர)ப் பரீட்சை - 2022(2023)

10 - இணைந்த கணிதம் I

புள்ளி வழங்கும் திட்டம்

பகுதி I

பகுதி A	= 10×25	=	**250**
பகுதி B	= 05×150	=	**750**

மொத்தம் = **1000/10**

இறுதிப் புள்ளி = 100

Common Techniques of Marking Answer Scripts.

It is compulsory to adhere to the following standard method in marking answer scripts and entering marks into the mark sheets.

1. Use a red color ball point pen for marking. (Only Chief/Additional Chief Examiner may use a mauve color pen.)
2. Note down Examiner's Code Number and initials on the front page of each answer script.
3. Write off any numerals written wrong with a clear single line and authenticate the alterations with Examiner's initials.
4. Write down marks of each subsection in a △ and write the final marks of each question as a rational number in a □ with the question number. Use the column assigned for Examiners to write down marks.

Example: **Question No. 03**

(i) .... √ △ $\frac{4}{5}$

(ii) .... √ △ $\frac{3}{5}$

(iii) .... √ △ $\frac{3}{5}$

(03) (i) $\frac{4}{5}$ + (ii) $\frac{3}{5}$ + (iii) $\frac{3}{5}$ = □ $\frac{10}{15}$

MCQ answer scripts: (Template)

1. Marking templets for G.C.E.(A/L) and GIT examination will be provided by the Department of Examinations itself. Marking examiners bear the responsibility of using correctly prepared and certified templates.
2. Then, check the answer scripts carefully. If there are more than one or no answers Marked to a certain question write off the options with a line. Sometimes candidates may have erased an option marked previously and selected another option. In such occasions, if the erasure is not clear write off those options too.
3. Place the template on the answer script correctly. Mark the right answers with a 'V' and the wrong answers with a 'X' against the options column. Write down the number of correct answers inside the cage given under each column. Then, add those numbers and write the number of correct answers in the relevant cage.

Structured essay type and assay type answer scripts:

1. Cross off any pages left blank by candidates. Underline wrong or unsuitable answers. Show areas where marks can be offered with check marks.

2. Use the right margin of the overland paper to write down the marks.
3. Write down the marks given for each question against the question number in the relevant cage on the front page in two digits. Selection of questions should be in accordance with the instructions given in the question paper. Mark all answers and transfer the marks to the front page, and write off answers with lower marks if extra questions have been answered against instructions.

4. Add the total carefully and write in the relevant cage on the front page. Turn pages of answer script and add all the marks given for all answers again. Check whether that total tallies with the total marks written on the front page.

Preparation of Mark Sheets.

Except for the subjects with a single question paper, final marks of two papers will not be calculated within the evaluation board this time. Therefore, add separate mark sheets for each of the question paper. Write paper 01 marks in the paper 01 column of the mark sheet and write them in words too. Write paper II Marks in the paper II Column and wright the relevant details.

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1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n}\frac{1}{r(r+1)}=\frac{n}{n+1}$ for all $n\in\mathbb{Z}^{+}$.

$n=1$, இற்கு L.H.S. $=\frac{1}{2}$, R.H.S. $=\frac{1}{2}$.

$\therefore n=1$. இற்கு முடிவு உண்மையாகும் (5)

ஏதாவது $k\in\mathbb{Z}^{+}$ இனை எடுக்க. $n=k$ இற்கு முடிவ உண்மை என்க.

i.e. $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$. .... (1) (5)

இப்போது

$$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\sum_{r=1}^{k}\frac{1}{r(r+1)}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)} \quad (5)$$

$$=\frac{k(k+2)+1}{(k+1)(k+2)}$$

$$=\frac{(k+1)^2}{(k+1)(k+2)}$$

$$=\frac{k+1}{k+2}. \quad (5)$$

எனவே $n=k$, இற்கு முடிவு உண்மையெனின் $n=k+1$. இற்கு முடிவு உண்மை.

$n=1$. இற்கு முடிவ உண்மை என காட்டியுள்ளோம்

ஆகவே கணித தொகுத்தறி முறையின் படி எல்லா $n\in\mathbb{Z}^{+}$. இற்கும் முடிவு உண்மையாகும் (5)

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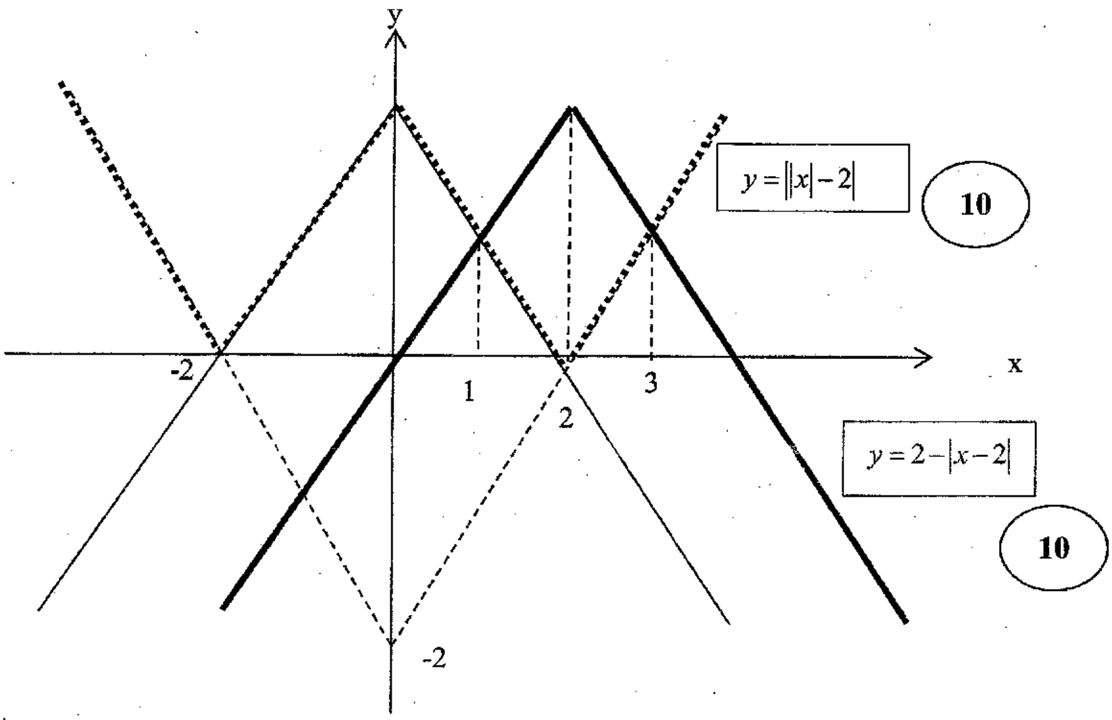
2. Sketch the graphs of $y = 2 - |x-2|$ and $y = ||x| - 2|$ in the same diagram.
Hence or otherwise, find all real values of x satisfying the inequality $||x| - 2| + |x-2| \leq 2$.



$||x| - 2| + |x-2| \leq 2$

$\Leftrightarrow ||x| - 2| \leq 2 - |x-2|$

வரைபிலிருந்து $1 \leq x \leq 3$. (5)

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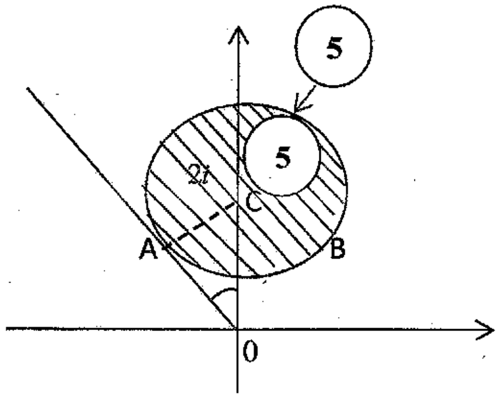
3. Shade in an Argand diagram, the region consisting of points that represent the complex numbers z satisfying the inequality $|\bar{z}+2i| \leq 1$.
Find the greatest value of Arg z for the complex numbers z represented by the points in this shaded region.

$|\bar{z}+2i| = |z-2i|$. (5)

தரப்பட்ட பிரதேசம் $|z-2i| \leq 1$. இனால் தரப்படும் பிரதேசத்தை ஒத்தது.



A. என்ற புள்ளியால் தரப்படும் சிக்கல் எண் z_0 என்க

ΔOAC, இலிருந்து $A\hat{O}C = \frac{\pi}{6}$. (5)

Arg z இன் தேவையான உயர பெறுமானம் $=$ Arg z_0

$$= \frac{\pi}{2} + \frac{\pi}{6}$$

$$= \frac{2\pi}{3}.$$ (5)

முதல் (5) க்கான வேறு முறை:

$z = x + iy$, என்க. இங்கு $x, y \in \mathbb{R}$

$$|\bar{z}+2i|^2 = |x-(y-z)i|^2$$ (5)

$$= x^2 + (y-2)^2$$

தரப்பட்ட பிரதேசம் $x^2 + (y-2)^2 \leq 1$. இனால் தரப்படும் பிரதேசத்தை ஒத்தது

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4. Let $a \in \mathbb{R}$. Write down the expansion of $(2+ax)^5$ in ascending powers of x up to and including x^2 term. **Hence**, find the values of a for which the coefficient of x^2 in the expansion of $(4-5x)(2+ax)^5$ is -80.

தேவையான விரிவு $= {}^5C_0 2^5 + {}^5C_1 2^4(ax) + {}^5C_2 2^3(ax)^2$ (5)

$= 32 + 5 \times 16ax + 10 \times 8a^2x^2$ (5)

$= 32 + 80ax + 80a^2x^2$

இப்போது, $(4-5x)(2+ax)^5 = 4(2+ax)^5 - 5x(2+ax)^5$

x^2 இன் குணகம் $= 4 \times 80a^2 - 5 \times 80a$ (5)

இது $4 \times 80a^2 - 5 \times 80a = -80$ இனால் தரப்படும்

(5)

$\therefore\ 4a^2 - 5a + 1 = 0.$

$\therefore\ (4a-1)(a-1) = 0.$

$\therefore\ a = \frac{1}{4}$ or $a = 1.$ (5)

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5. Show that $\lim_{x \to 0} \frac{x((1+x)\operatorname{cosec} 2x - \cot 2x)}{\sqrt{1+2x}-\sqrt{1-2x}} = \frac{1}{4}$.

$$\lim_{x\to 0}\frac{x\left((1+x)\operatorname{cosec}2x-\cot 2x\right)}{\left(\sqrt{1+2x}-\sqrt{1-2x}\right)}$$

$$=\lim_{x\to 0}\frac{x}{\sin 2x}\cdot\frac{(1+x-\cos 2x)}{\left(\sqrt{1+2x}-\sqrt{1-2x}\right)}$$ (5)

$$=\lim_{x\to 0}\frac{x}{\sin 2x}\cdot\frac{(1+x-\cos 2x)}{\left(\sqrt{1+2x}-\sqrt{1-2x}\right)}\times\frac{\left(\sqrt{(1+2x)}+\sqrt{1-2x}\right)}{\left(\sqrt{1+2x}+\sqrt{1-2x}\right)}$$ (5)

$$=\lim_{x\to 0}\frac{1}{2}\cdot\frac{2x}{\sin 2x}\cdot\frac{\left(2\sin^2 x+x\right)}{\left[(1+2x)-(1-2x)\right]}\cdot\left(\sqrt{1+2x}+\sqrt{1-2x}\right)$$

$$=\lim_{x\to 0}\frac{1}{2}\cdot\frac{2x}{\sin 2x}\cdot\left(\frac{2\sin^2 x}{4x}+\frac{1}{4}\right)\left(\sqrt{1+2x}+\sqrt{1-2x}\right)$$ (5)

$$=\frac{1}{2}\times 1\times\frac{1}{4}\times 2$$ (10)

$$=\frac{1}{4}.$$

All three limits correct	(10)
Any two	(5)

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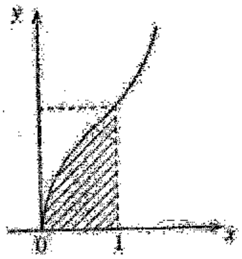
6. Using $\frac{d}{dx}\left\{x(x^2+1)\tan^{-1}x\right\}=(3x^2+1)\tan^{-1}x+x$, show that $\int_0^1 (3x^2+1)\tan^{-1}x\,dx=\frac{1}{2}(\pi-1)$.

The region enclosed by the curves $y=\sqrt{2(3x^2+1)\tan^{-1}x}$, $x=1$ and $y=0$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\pi(\pi-1)$.



$\frac{d}{dx}\left\{(x^2+1)\tan^{-1}x\right\}=(3x^2+1)\tan^{-1}x+x,$ இனைப் பயன்படுத்தி

$\int_0^1\left[(3x^2+1)\tan^{-1}x+x\right]dx = x(x^2+1)\tan^{-1}x\Big|_0^1$ (5)

$\therefore \int_0^1 (3x^2+1)\tan^{-1}x\,dx + \int_0^1 x\,dx = 2\tan^{-1}1$

$\therefore \int_0^1 (3x^2+1)\tan^{-1}x\,dx + \frac{x^2}{2}\Big|_0^1 = 2\frac{\pi}{4}$ (5)

$\therefore \int_0^1 (3x^2+1)\tan^{-1}x\,dx = \left(\frac{\pi}{2}-\frac{1}{2}\right)$

$=\frac{1}{2}(\pi-1).$ (5)

தேவையான கனவளவு $=\pi\int_0^1 2\left(3x^2+1\right)\tan^{-1}x\;dx$ (5)

$=2\pi\frac{1}{2}(\pi-1)$ (5)

$=\pi(\pi-1).$

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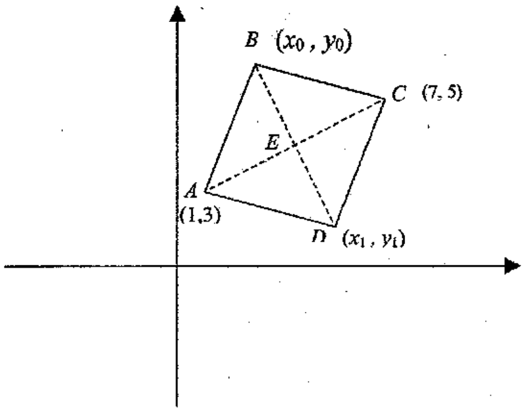
8. Let $ABCD$ be a square with $A \equiv (1, 3)$ and $C \equiv (7, 5)$. Find the x-coordinates of B and D.



$B = (x_0, y_0), D = (x_1, y_1)$ என்க

Since E in the mid-point of AC, we have $E \equiv (4, 4)$. (5)

எனின் $AE^2 = 3^2+1^2 = 10$

$ABCD$ ஆனது சதுரம் ஆகையால் $BE = AE$.

எனவே Hence, $(x_0 - 4)^2 + (y_0 - 4)^2 = 10.$ ---------------- (1) (5)

Also, $AE \perp BE$.

$\therefore \left(\frac{4-3}{4-1}\right) \times \left(\frac{y_0 - 4}{x_0 - 4}\right) = -1.$ (5)

Hence, $y_0 - 4 = -3(x_0 - 4)$ ---------------- (2)

(1), (2) $\Rightarrow (x_0 - 4)^2 + 9(x_0 - 4)^2 = 10.$ (5)

Hence, $y_0 - 4 = -3(x_0 - 4)$.

$\therefore \ (x_0 - 4)^2 = 1.$

$\therefore \ (x_0 - 4) = \pm 1.$

$\therefore \ x_0 = 5 \ \text{ or } \ x_0 = 3.$ (5)

Note that (x_1, y_1) also satisfies (1) and (2), when (x_0, y_0) is replaced by (x_1, y_1).

Hence, x coordinates of B and D are 3 and 5.

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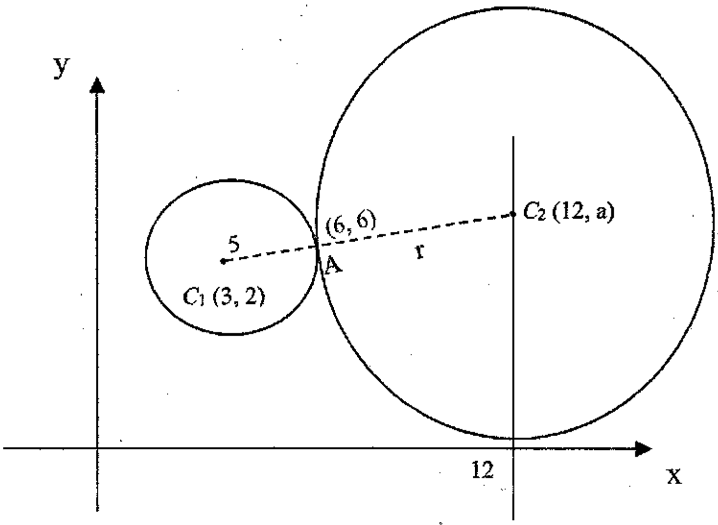
9. Find the equation of the circle that touches the circle $x^2+y^2-6x-4y-12=0$ externally at the point (6, 6) and has its centre on the line $x=12$.



தரப்பட்ட வட்டத்தின் மையம் C_1 என்க தேவையான வட்டத்தின் மையம் C_2 என்க

Then $C_1 \equiv (3,2)$, $C_2 \equiv (12,\ a)$; where $a \in \mathbb{R}$ (5)

Since the circles touch externally C_2 lies on the line C_1A.

$\therefore \dfrac{6-2}{6-3} = \dfrac{a-6}{12-6}.$ (5)

$\therefore 3a - 18 = 24.$

$\therefore a = 14.$ (5)

The radius of the required circle $C_2 = \sqrt{(12-6)^2 + (14-6)^2}$

$= 10.$ (5)

Hence, the required equalion is $(x-12)^2 + (y-14)^2 = 100.$ (5)

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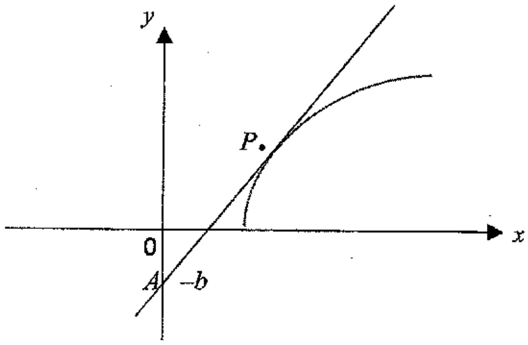
7. Let $a, b > 0$. A curve is parametrically given by $x = a\sec\theta$ and $y = b\tan\theta$ for $0 < \theta < \frac{\pi}{2}$. The tangent line to the curve at the point $P \equiv (a\sec\theta,\ b\tan\theta)$ passes through the point $(0, -b)$. Find the coordinates of P.



$x = \text{a}\sec\theta,\qquad y = b\tan\theta$

$\frac{dx}{d\theta} = a\sec\theta\tan\theta,\quad \frac{dy}{d\theta} = b\sec^2\theta$ (5)

$\therefore \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$ (5)

$\therefore \quad = \frac{b\sec\theta}{a\tan\theta}.$

AP இன் படித்திறன் $AP = \frac{b + b\tan\theta}{a\sec\theta}.$

தரப்பட்ட நியந்தனையிலிருந்து $\frac{b\sec\theta}{a\tan\theta} = \frac{b(1+\tan\theta)}{a\sec\theta}.$ (5)

$\therefore\ \sec^2\theta = \tan\theta + \tan^2\theta$

$\therefore\ \tan\theta = 1$ (5)

$\therefore\ \theta = \frac{\pi}{4}$

$\therefore\ P \equiv \left(\sqrt{2}a,\ b\right)$ (5)

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10. Show that $\cos 5\theta = \cos 3\theta$ if and only if $\theta = \frac{n\pi}{4}$ for $n \in \mathbb{Z}$.

Show also that $\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta - \cos 3\theta} = -\cot 4\theta$ for $\theta \neq \frac{n\pi}{4}$ and $n \in \mathbb{Z}$.

$\cos 5\theta = \cos 3\theta$

$\Leftrightarrow \quad 5\theta = 2n\pi \pm 3\theta$ for $n \in \mathbb{Z}$. (5)

$\Leftrightarrow \quad 8\theta = 2n\pi$ or $2\theta = 2n\pi$ for $n \in \mathbb{Z}$.

$\Leftrightarrow \quad \theta = \frac{n\pi}{4}$ or $\theta = n\pi$ for $n \in \mathbb{Z}$. (5)

$\Leftrightarrow \quad \theta = \frac{n\pi}{4}$ for $n \in \mathbb{Z}$.

$$\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta - \cos 3\theta} = \frac{2\cos 4\theta \sin\theta}{-2\sin 4\theta \sin\theta}$$ (5) (5)

$$= -\cot 4\theta$$ (5)

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Part B

* Answer five questions only.

11. (a) Let $0<|p|<1$. Show that the equation $p^2x^2-2x+1=0$ has real distinct roots.
Let α and β $(>\alpha)$ be these roots. Show that α and β are both positive.
Find $(\alpha-1)(\beta-1)$ in terms of p, and deduce that $\alpha<1$ and $\beta>1$.
Show that $\sqrt{\beta}-\sqrt{\alpha}=\frac{1}{|p|}\sqrt{2(1-|p|)}$

It is given that $\sqrt{\beta}+\sqrt{\alpha}=\frac{1}{|p|}\sqrt{2(1+|p|)}$. Show that the quadratic equation whose roots are $|\sqrt{\alpha}-1|$ and $|\sqrt{\beta}-1|$ is $|p|x^2-\sqrt{2(1-|p|)}\,x+\sqrt{2(1+|p|)}-|p|-1=0$.

(b) Let $p(x)=2x^3+ax^2+bx-4$, where $a, b\in\mathbb{R}$. It is given that $(x+2)$ is a factor of both $p(x)$ and $p'(x)$, where $p'(x)$ is the derivative of $p(x)$ with respect to x. Find the values of a and b. For these values of a and b, completely factorise $p(x)-3p'(x)$.

(a)

$0<|p|<1$.

$p^2x^2-2x+1=0$. இன் பிரித்துக்காட்டி Δ என்க

$p^2<1$. ஆதலால் $\therefore \Delta=4-4p^2=4(1-p^2)>0,$ (5) (5)

$\therefore$ சமன்பாடு இரு வேறுவேறான மெய் மூலங்களை கொண்டிருக்கும் (5) **15**

$\alpha,\ \beta\ (>\alpha)$ ஆகியன மூலகங்கள் என்க.

எனின் $\alpha\beta=\frac{1}{p^2}>0.$ (5)

α and β இரண்டும் நேரானவை அல்லது மறையானவை.

எனினும் $\alpha+\beta=\frac{2}{p^2}>0$ ஆதலால் $\alpha,\ \beta$ இரண்டும் நேரானது. (5) (5) **15**

$(\alpha-1)(\beta-1)=\alpha\beta-(\alpha+\beta)+1=\frac{1}{p^2}-\frac{2}{p^2}+1=\frac{p^2-1}{p^2}<0$ and $\alpha-1<\beta-1.$ (5) (5) (5)

$\therefore \alpha-1<0$ and $\beta-1>0.$ (5)

$\therefore\ \alpha<1$ and $\beta>1.$ **20**

$$(\sqrt{\beta}-\sqrt{\alpha})^2=\alpha+\beta-2\sqrt{\alpha\beta}=\frac{2}{p^2}-2\frac{1}{|p|}=\frac{2}{p^2}(1-|p|).$$ (5) (5)

$$\therefore \sqrt{\beta}-\sqrt{\alpha}=\frac{1}{|p|}\sqrt{2(1-|p|)}$$ (5)

15

தேவையான சமன்பாடு $\left(x-\left|\sqrt{\alpha}-1\right|\right)\left(x-\left|\sqrt{\beta}-1\right|\right)=0.$ (10)

$$x^2-\left(\left|\sqrt{\alpha}-1\right|+\left|\sqrt{\beta}-1\right|\right)x+\left|\sqrt{\alpha}-1\right|\left|\sqrt{\beta}-1\right|=0$$

$\left|\sqrt{\alpha}-1\right|=1-\sqrt{\alpha}$, $\left|\sqrt{\beta}-1\right|=\sqrt{\beta}-1$ ஆதலால்

$$x^2-(\sqrt{\beta}-\sqrt{\alpha})x+\sqrt{\alpha}+\sqrt{\beta}-\sqrt{\alpha\beta}-1=0$$ (5)

$$\therefore\ x^2-\frac{1}{|p|}\sqrt{2(1-|p|)}\,x+\frac{1}{|p|}\sqrt{2(1+|p|}-\frac{1}{|p|}-1=0$$

$$\therefore\ |p|x^2-\sqrt{2(1-|p|}\,x+\sqrt{2(1+|p|}-|p|-1=0$$ (5)

20

$$p(x)=2x^3+ax^2+bx-4$$

$\therefore p'(x)=6x^2+2ax+b.$ (5)

$(x+2)$ ஆனது $p\ (x)$, இன் காரணி ஆதலால்

$p(-2)=0.$ (5)

இப்போது, $p(-2)=-16+4a-2b-4=0.$ (5)

$\therefore\ 2a-b=10$ ----------------- (1)

$(x+2)$ ஆனது $p'(x)$, இன் காரணி ஆதலால்

$p'(-2)=0.$ (5)

இப்போது, $p'(-2)=24-4a+b=0.$ (5)

$\therefore 4a-b=24.$ ----------------- (2)

(1) and (2) $\Rightarrow a = 7$ and $b = 4$.

(5) (5)

35

$$p(x) - 3p'(x) = (2x^3 + 7x^2 + 4x - 4) - 3(6x^2 + 14x + 4)$$ (5)

$$= (x+2)(2x^2 + 3x - 2) - 3(x+2)(6x+2)$$ (5)

$$= (x+2)\left[2x^2 + 3x - 2 - 18x - 6\right]$$

$$= (x+2)\ (2x^2 - 15x - 8)$$ (5)

$$= (x+2)(2x+1)(x-8)$$

(5) (5) (5)

30

வேறு முறை:

$p(x) = 2x^3 + ax^2 + bx - 4$

$(x+2)$ ஆனது $p(x)$, $p'(x)$ இரண்டினதும் காரணி ஆகையால்

$p(x) = (x+2)^2(2x+k)$. (5) இங்கு k ஒரு மாறிலி.

(10)

மாறிலிகளை ஒப்பிட $4k = -4$

$\therefore k = -1$ (5)

$\therefore p(x) = (x+2)^2(2x-1)$.

$\therefore p(x) = (x^2 + 4x + 4)(2x-1) = 2x^3 + 7x^2 + 4x - 4$. (5)

x: இன் குணங்களை ஒப்பிட $b = 4$ and $a = 7$.

(5) (5)

$\therefore p(x) = 2x^3 + 7x^2 + 4x - 4$

$\therefore p'(x) = 6x^2 + 14x + 4 = 2(3x^2 + 7x + 2) = 2(x+2)(3x+1)$ (5)

$\therefore p(x) - 3p'(x) = (x+2)^2(2x-1) - 3(2(x+2)(3x+1))$ (5)

$= (x+2)[(x+2)(2x-1) - 6(3x+1)]$

$= (x+2)(2x^2 - 15x - 8)$ (5)

$= (x+2)(2x+1)(x-8)$ (5)

(5) (5)

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12. (*a*) Six mangoes and four oranges are to be distributed among eight students so that each student receives at least one fruit.

Find the number of different ways in which

(i) six students get one fruit each and out of the remaining two students one gets **two mangoes** and the other gets **two oranges**,

(ii) seven students get one fruit each, and the other student gets **three mangoes**,

(iii) seven students get one fruit each, and the other student gets **three fruits**.

(*b*) Let $U_r = \frac{4(2r+7)}{(2r+1)(2r+3)(2r+5)}$ for $r \in \mathbb{Z}^+$. Also, let $f(r) = \frac{A}{(2r+1)} + \frac{B}{(2r+3)}$ for $r \in \mathbb{Z}^+$, where A and B are real constants. Determine the values of A and B such that $U_r = f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$.

Hence or otherwise, show that $\sum_{r=1}^{n} U_r = \frac{4}{5} - \frac{3}{2n+3} + \frac{1}{2n+5}$ for $n \in \mathbb{Z}^+$.

Deduce that the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Hence, find the value of the real constant k such that $\sum_{r=1}^{\infty} (U_r + kU_{r+1}) = 1$.

(*a*) (i)

2 மாணவர்கள்		6 மாணவர்கள்
(2M) (2OR)		(4M) (2OR)
${}^{8}C_2$	X	${}^{6}C_4 \times {}^{2}C_2$
(2OR) (2M)		(4M) (2OR)
${}^{8}C_2$	X	${}^{6}C_4 \times {}^{2}C_2$

(5) (5)

தேவையான வழிகள் : $2 \times {}^{8}C_2 \times {}^{6}C_4 \times {}^{2}C_2$ (5)

$= 2 \times \frac{8!}{6!2!} \times \frac{6!}{4!2!} = 2 \times 28 \times 15 = 840.$ (5)

(5)

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(3M) ... (3M) (4OR)

$^8C_1$ x $^7C_3 \times {}^4C_4$

தேவையான வழிகள்: $^9C_1 \times {}^7C_3 \times {}^4C_4 = 8 \times 4\frac{7!}{4!3!} = 8 \times 35 = 280$. (5) (5) (5)

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(iii) 3- பழம் : (3M) + (3OR) (2M) + (1OR) (1M) + (2OR) (5)

4 வகைகள்

(3M) வகை 280 வழிகள் (ii) இல் உள்ளவாறு

(3OR) வகை $^8c_1 \times {}^7c_6 \times {}^1c_1 = 8 \times 7 = 56$ வழிகள் (5)

(2M) + (1OR) வகை $^8c_1 \times {}^7c_4 \times {}^3c_3 = 8 \times 35 = 280$ வழிகள் (5)

(1M) + (2OR) வகை $^8c_1 \times {}^7c_5 \times {}^2c_2 = 8 \times 21 = 168$ வழிகள் (5)

தேவையான வழிகள்= $280 + 56 + 280 + 168$

$= 784$ (5)

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வேறு முறை:

(a) மாம்பழங்கள் 6; தோடம்பழங்கள் 4; மாணவர்கள் 8.

(i) ஒரு மாணவனுக்கு 2 மாம்பழங்களும் ,இன்னொருவருக்கு 2 தோடம் பழங்களும் வழங்கப்படுவதால் 6 மாணவர்கள் 4 மாம்பழங்கள்,2 தோடம்பழங்களில் இருந்து ஒவ்வொன்றை பெறுவர்.

						2Ma	2Or

4 மாம்பழங்களும், 2 தோடம்பிங்களும் 6 மாணவர்களுகிடையில் (ஒன்று வீதம்) பங்கிடப்படும் வழிகள்

$= \frac{6!}{4!2!}$ (10)

8மாணவர்களிலிருந்து ஒரு மாணவர் தெரிவு செய்யப்பட்டு 2 மாம்பழங்களை வழங்குவதற்கான வழிகள் $= {}^{8}C_1$

7 மாணவர்களிலிருந்து மற்றுமொரு மாணவன் தெரிவு செய்யப்பட்டு 2 தோடம்பழங்களை வழங்குவதற்கான வழிகள் $= {}^{7}C_1$ (5)

தேவையான வழிகள் $= \frac{6!}{4!2!} \times {}^{8}C_1 \times {}^{7}C_1$

$= 840$ ways. (5)

OR

$= \frac{6!}{4!2!} \times {}^{8}P_2$

$= 840$ ways.

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(ii) 7 மாணவர்கள் ஒவ்வொரு பழம் வீதமும் ஒரு மாணவன் மூன்று மாம்பழங்களையும் பெறுகையில்

							3Ma

3மாம்பழங்களும் 4 தோடம்பிங்களும் ஆளுக்கு ஒவ்வொன்று வீதம் 7 மாணவர்களிடையே பகிந்தளிக்க கூடிய வழிகள் $= \frac{7!}{4!3!}$ (5)

எட்டு மாணவர்களிலிருந்து ஒரு மாணவன் தெரிவு செய்யப்பட்டு 3 மாம்பழங்களை வழங்குவதற்கான வழிகள் $= {}^{8}C_1$ (5)

$\therefore$ தேவையான வழிகள் $= {}^{8}C_1 \times \frac{7!}{4!3!}$ (5)

$= 280$ ways.

(iii)

3 பழங்கள் ஒரு மாணவனுக்கு வழங்கல		7 பழங்கள் 7 மாணவர்களுக்கு வழங்கல்		தேவையான வழிகள்	
மாம்பழம்	தோடம்பழம்	மாம்பழம்	தோடம்பழம்		
3	0	3	4	$= {}^{8}C_1 \times \frac{7!}{3!4!} = 280$	(5)
2	1	4	3	$= {}^{8}C_1 \times \frac{7!}{4!3!} = 280$	(5)

1	2	5	2	$= {}^8C_1 \times \frac{7!}{5!2!} = 168$	5
0	3	6	1	$= {}^8C_1 \times \frac{7!}{6!} = 56$	5

தேவையான வழிகள்

$= 280 + 280 + 168 + 56$

$= 784$ (5)

25

(*b*). $r \in \mathbb{Z}^+$

$$U_r = \frac{4(2r+7)}{(2r+1)(2r+3)(2r+5)}$$

$$U_r = f(r) - f(r+1)$$

$$\frac{4(2r+7)}{(2r+1)(2r+3)(2r+5)} = \frac{A}{2r+1} + \frac{B}{2r+3} - \frac{A}{2r+3} - \frac{B}{2r+5}$$ (5)

$$\therefore 4(2r+7) = A(2r+3)(2r+5) + (B-A)(2r+1)(2r+5) - B(2r+1)(2r+3)$$

$$= (4A+4B)r + 10A - 2B$$

Any Method (10)

r இன் அடுக்குகளின் குணங்களை ஒப்பிட

r : $8 = 4A + 4B \Rightarrow 2 = A + B$

r^0 : $28 = 10A + 2B \Rightarrow 14 = 5A + B$

$A = 3, \quad B = -1$ (5) (5)

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$U_r = f(r) - f(r+1)$ இங்கு $f(r) = \frac{3}{2r+1} - \frac{1}{2r+3}$ (5)

$r = 1;\ U_1 = f(1) - f(2)$

$r = 2;\ U_2 = f(2) - f(3)$ (5)

$\vdots \quad \vdots$

$r = n-1;\ U_{n-1} = f(n-1) - f(n)$

(5)

$r = n;\quad U_n = f(n) - f(n+1)$

$$\sum_{r=1}^{n} U_r = f(1) - f(n+1) \quad (5)$$

$$\therefore \sum_{r=1}^{n} U_r = f(1) - f(n+1) \quad (5)$$

$$= 1 - \frac{1}{5} - \frac{3}{2n+3} + \frac{1}{2n+5}$$

$$= \frac{4}{5} - \frac{3}{2n+3} + \frac{1}{2n+5} \quad (5) \quad r \in \mathbb{Z}$$

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$$\lim_{n \to \infty} \sum_{r=1}^{n} U_r \quad (5)$$

$$= \lim_{n \to \infty} \left(\frac{4}{5} - \frac{3}{2n+3} + \frac{1}{2n+5} \right)$$

$$= \frac{4}{5} \quad (5)$$

$\therefore$ முடிவிலி தொடர் $\sum_{r=1}^{\infty} U_r$ ஆனது ஒருங்கும் அதன் கூட்டுத்தொகை $\frac{4}{5}$. (5)

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$$1 = \sum_{r=1}^{\alpha} (U_r + kU_{r+1})$$

$$= (1+k)\left(\sum_{r=1}^{\alpha} U_r \right) - kU_1 \quad (5)$$

$$= (1+k)\left(\frac{4}{5} \right) - k\left(\frac{12}{35} \right) \quad (5)$$

$$\therefore k = \frac{7}{16} \quad (5)$$

15

13.(a) Let $\mathbf{A} = \begin{pmatrix} a & -2 \\ 1 & a+2 \end{pmatrix}$. Show that $\mathbf{A}^{-1}$ exists for all $a \in \mathbb{R}$.

The matrices $\mathbf{P} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 7 & 4 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ are such that $\mathbf{A} = \mathbf{P}\mathbf{Q}^{\mathrm{T}} + \mathbf{R}$. Show that $a = 1$.

For this value of a, write down $\mathbf{A}^{-1}$ and hence, find the values of x and y such that $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$.

(b) Let $z, w \in \mathbb{C}$. Show that $z\bar{z} = |z|^2$ and hence, show that $|z+w|^2 = |z|^2 + 2\,\mathrm{Re}(z\bar{w}) + |w|^2$.

Deduce that $|z+w|^2 + |z-w|^2 = 2\left(|z|^2 + |w|^2\right)$ and give a geometric interpretation for it when the points representing z, w and 0 in the Argand diagram are non-collinear.

(c) Let $z = -1 + \sqrt{3}i$. Express z in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $\frac{\pi}{2} < \theta < \pi$.

Let $z^n = a_n + ib_n$, where $a_n, b_n \in \mathbb{R}$ for $n \in \mathbb{Z}^+$. Write down $\mathrm{Re}\left(z^m \cdot z^n\right)$ in terms of a_m, a_n, b_m and b_n for $m, n \in \mathbb{Z}^+$.

Considering z^{m+n} and using De Moivre's theorem, show that $a_m a_n - b_m b_n = 2^{m+n}\cos(m+n)\frac{2\pi}{3}$, for $m, n \in \mathbb{Z}^+$.

(a) $|A| = a(a+2) + 2 = a^2 + 2a + 2 = (a+1)^2 + 1 \neq 0$ for all $a \in \mathbb{R}$. (5) (5)

$\therefore$ எல்லா $a \in \mathbb{R}$ இற்கு A^{-1} உண்டு (5)

15

$A = PQ^T + R$ (5)

$$\begin{pmatrix} \alpha & -2 \\ 1 & a+2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 7 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$ (5) (5)

$$= \begin{pmatrix} 0 & -5 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix}$$ (5)

$a = 1$ and $a + 2 = 3$. $\therefore a = 1$ (5)

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When $a=1$, $A=\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}$ $\therefore A^{-1}=\frac{1}{5}\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}$ (10)

$A\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-5\\ 10\end{pmatrix}$

$\therefore$ $\begin{pmatrix}x\\ y\end{pmatrix}=A^{-1}\begin{pmatrix}-5\\ 10\end{pmatrix}=\frac{1}{5}\begin{pmatrix}3 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}-5\\ 10\end{pmatrix}=\begin{pmatrix}1\\ 3\end{pmatrix}$ (10)

$\therefore$ $x=1$ (5) and $y=3$ (5).

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(*b*) Taking $z=x+iy$; $x,y\in\mathbb{R}$,

$z\overline{z}=(x+iy)(x-iy)=x^2-i^2y^2=x^2+y^2=|z|^2$

(5) (5)

10

$|z+w|^2=(z+w)\overline{(z+w)}$ (5)

$=(z+w)(\overline{z}+\overline{w})$ (5)

$=z\overline{z}+z\overline{w}+\overline{z}w+w\overline{w}$ (5)

$=|z|^2+z\overline{w}+\overline{z\overline{w}}+|w|^2$

$=|z|^2+2\,\mathrm{Re}(z\overline{w})+|w|^2$ (5)

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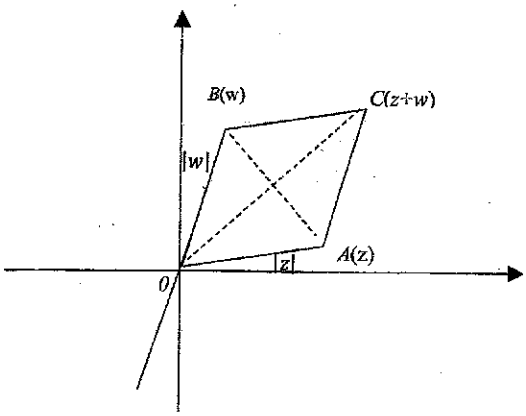
Note that $|z-w|^2=|z|^2-2\,\mathrm{Re}(z\overline{w})+|w|^2$ by (5)

$\therefore$ $|z+w|^2+|z-w|^2=2\left(|z|^2+|w|^2\right)$ (5)



B

If z, w and 0 are non-collinear, then $OC^2 + AB^2 = 2\left(OA^2 + OB^2\right)$.

$\left(\because OC = |z+w| \text{ and } AB = |z+w|.\right)$

ஒரு இணைகரத்தில் மூலை விட்டங்களின் வர்க்கங்களின் கூட்டுத் தொகையானது அதன் பக்க நீளங்களின் வர்க்கங்களின் கூட்டுத் தொகைக்கு சமனாகும்

.

(5)

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(c) $z = -1 + \sqrt{3}i = 2\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right) = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ (10)

(5)

இங்கு $r = 2$, and $\theta = \frac{2\pi}{3}$.

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(5)

$\text{Re}(z^m z^n) = \text{Re}\left[\left(a_m + ib_m\right)\left(a_n + ib_n\right)\right] = a_m a_n - b_m b_n$ -------- (1)

05

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(5) (5)

$$z^m z^n = z^{m+n} = \left[2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]^{n+m} = 2^{m+n}\left[\cos\frac{2(m+n)\pi}{3} + i\sin\frac{2(m+n)\pi}{3}\right]$$

$$\therefore \text{Re}(z^m z^n) = 2^{m+n}\cos(m+n)\frac{2\pi}{3}$$ ------------------ (2)

(5)

(1) and (2) $\Rightarrow a_m a_n - b_m b_n = 2^{m+n}\cos(m+n)\frac{2\pi}{3}$.

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14. (*a*) Let $f(x)=\dfrac{2x+3}{(x+2)^2}$ for $x\neq -2$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{-2(x+1)}{(x+2)^3}$ for $x\neq -2$.

Hence, find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Also, find the coordinates of the turning point of $f(x)$.

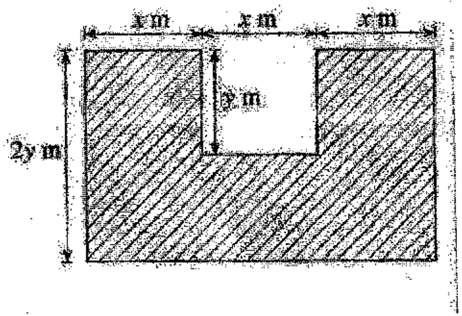
It is given that $f''(x)=\dfrac{2(2x+1)}{(x+2)^4}$ for $x\neq -2$. Find the coordinates of the point of inflection of the graph of $y=f(x)$.

Sketch the graph of $y=f(x)$ indicating the asymptotes, the turning point and the point of inflection.

State the smallest value of k for which $f(x)$ is one-one on $[k,\infty)$.

(*b*) The shaded region shown in the figure is of area 45 m^2. It is obtained by removing a rectangle of length x m and width y m from a rectangle of length $3x$ m and width $2y$ m. Show that the perimeter L m of the shaded region is given by $L=6x+\dfrac{54}{x}$ for $x>0$.

Find the value of x such that L is minimum.



(a) For $x\neq -2$, ஆக $f(x)=\dfrac{2x+3}{(x+2)^2}$.

$$f'(x)=\frac{(x+2)^2(2)-2(2x+3)(x+2)}{(x+2)^4} \quad (20)$$

$$=\frac{2(x+2)[x+2-2x-3]}{(x+2)^4}$$

$$=\frac{-2(x+1)}{(x+2)^3} \quad (5)$$

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$f'(x)=0 \Leftrightarrow x=-1$ (5)

	$-\infty<x<-2$	$-2<x<-1$	$-1<x<\infty$
$f'(x)$ இன் குறி	(−)	(+)	(−)
$f(x)$ is	குறைவடைகின்றது ↘	அதிகரிக்கின்றது ↗	குறைவடைகின்றது ↘
	(5)	(5)	(5)

$\therefore f(x)$ ஆனது $(-2,-1]$ and இல் அதிகரிக்கின்றது அத்துடன்

$(-\infty,-2)$, $[-1,\infty)$ இல் குறைவடைகின்றது

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திரும்பற் புள்ளி $(-1,1)$ ஆனது ஒரிட உயர்வாகும்

5 5

$$f''(x)=\frac{2(2x+1)}{(x+2)^4}$$

$f''(x)=0 \Leftrightarrow x=\frac{-1}{2}.$ 5

	$-2<x<-\frac{1}{2}$	$-\frac{1}{2}<x<\infty$
$f''(x)$ இன் குறி	$(-)$	$(+)$
குவிவுத்தன்மை	கீழ்நோக்கி குவிந்தது	மேல்நோக்கி குவிந்தது

5 5

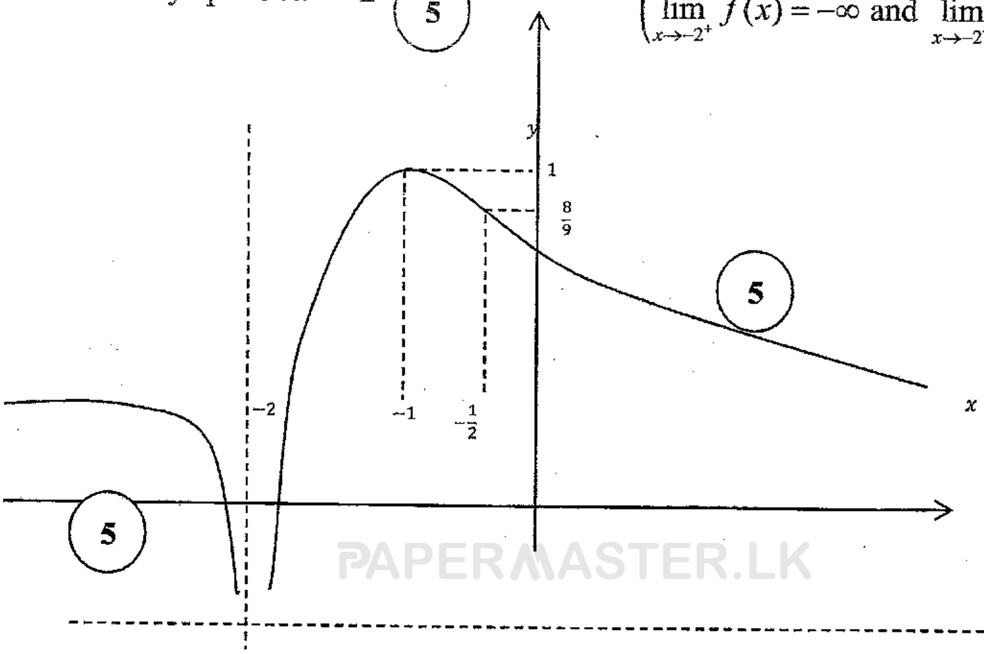
$\therefore$ the point of inflection is $\left(\frac{-1}{2},\frac{8}{9}\right)$ விபத்தி புள்ளியாகும்

5

x - intercept: $\left(-\frac{3}{2},0\right)$ 5

Horizontal Asymptote: $\lim_{x\to\pm\infty} f(x)=0$ 5 $\therefore y=0$

Vertical Asymptote : $x=-2$ 5 $\left(\lim_{x\to-2^+} f(x)=-\infty \text{ and } \lim_{x\to-2^-} f(x)=-\infty\right)$



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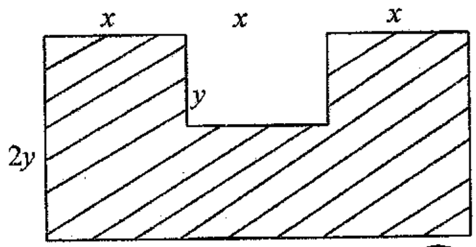
$f(x)$ ஆனது $[k,\infty)$ இன்மேல் ஒன்றுக்கொன்றானது ஆக k, இன் மிகக்குறைந்த பொறுமானம $k=-1$. (5)

05

(b)



for $x>0$, $y>0$

(5) (5)

நிழற்றிய பிரதேசத்தின் பரப்பு $45=(3x)(2y)-xy$

$\therefore\ 45=5xy$

$\therefore\ y=\frac{9}{x}$ (5)

$L=6x+6y$ (10)

$=6x+\frac{54}{x}$ for $x>0$ (5)

$\frac{dL}{dx}=6-\frac{54}{x^2}=\frac{6(x^2-9)}{x^2}=\frac{6(x-3)(x+3)}{x^2}$ (5)

$\frac{dL}{dx}=0 \quad \Leftrightarrow \quad x=3$ (5)

For $0<x<3$, $\frac{dL}{dx}<0$ and

For $x>3$, $\frac{dL}{dx}>0$. (5)

$\therefore$ L is minimum when $x=3$.

45

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15. (a) Find the values of the constants A, B and C such that

$x^2+x+2=A(x^2+x+1)+(Bx+C)(x+1)$ for all $x\in\mathbb{R}$.

Hence, write down $\dfrac{x^2+x+2}{(x^2+x+1)(x+1)}$ in partial fractions and find $\displaystyle\int\frac{x^2+x+2}{(x^2+x+1)(x+1)}\,dx$.

(b) Show that $1+\sin 2x=2\cos^2\left(\frac{\pi}{4}-x\right)$ and hence, show that $\displaystyle\int_0^{\frac{\pi}{2}}\frac{1}{1+\sin 2x}\,dx=1$.

(c) Let $I=\displaystyle\int_0^{\frac{\pi}{2}}\frac{x^2\cos 2x}{(1+\sin 2x)^2}\,dx$. Using integration by parts, show that $I=-\dfrac{\pi^2}{8}+J$, where $J=\displaystyle\int_0^{\frac{\pi}{2}}\frac{x}{1+\sin 2x}\,dx$.

Using the relation $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ and the result in (b), evaluate J and show that $I=\dfrac{\pi}{8}(2-\pi)$.

(a)

$$x^2+x+2=A(x^2+x+1)+(Bx+C)(x+1)$$
$$=(A+B)x^2+(A+B+C)x+A+C$$

x இனது அடுக்குகளின் குணகங்களை ஒப்பிட:

$x^0:\quad z=A+C$

$x:\quad 1=A+B+C$ (5)

$x^2:\quad 1=A+B$

$\therefore\ A=2,\quad B=-1\quad and\quad C=0.$ (5) (5) (5)

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$$\frac{x^2+x+2}{(x^2+x+1)(x+1)}=\frac{2}{x+1}-\frac{x}{x^2+x+1}$$ (5)

$$\therefore\int\frac{x^2+x+2}{(x^2+x+1)(x+1)}dx=2\int\frac{1}{x+1}dx-\int\frac{x}{x^2+x+1}dx$$ (5)

$$= 2\ln|x+1| - \frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx + \frac{1}{2}\int \frac{1}{x^2+x+1}dx$$ (5) (5) (5)

$$-\frac{1}{2}\ln(x^2+x+1) + \frac{1}{2}\frac{1}{\sqrt{\frac{3}{2}}}\tan^{-1}\frac{(x+\frac{1}{2})}{\sqrt{\frac{3}{2}}} + C$$ (5) (5) (5)

$x^2+x+1>0$

$$= 2\ln|x+1| - \frac{1}{2}\ln(x^2+x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{(2x+1)}{\sqrt{3}} + C,$$ where C is an arbitrary constant.

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(*b*)

$$2\cos^2(\frac{\pi}{4}-x) = 2(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x)^2$$

$$=(\cos x+\sin x)^2$$ (5)

$$=1+2\sin x\cos x$$ (5)

$$=1+\sin 2x$$ (5)

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$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin 2x}dx = \int_0^{\frac{\pi}{2}} \frac{1}{2\cos^2\left(\frac{\pi}{4}-x\right)}dx$$ (5)

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{\pi}{4}-x\right)dx$$ (5)

$$=\frac{-1}{2}\tan\left(\frac{\pi}{4}-x\right)\Bigg|_0^{\frac{\pi}{2}}$$ (5)

$$=\frac{-1}{2}\left(\tan\left(\frac{-\pi}{4}\right)-\tan\frac{\pi}{4}\right)$$ (5)

$$= \frac{-1}{2}(-1-1)$$

$$= 1 \quad (5)$$

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(C) $$I = \int_0^{\frac{\pi}{2}} \frac{x^2 \cos 2x}{(1+\sin 2x)^2} dx$$

$$= x^2 \left(\frac{-1}{2}\right) \frac{1}{1+\sin 2x} \Bigg|_0^{\frac{\pi}{2}} + \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x} dx \quad (5)$$

(5) (5)

$$= \frac{-1}{2} \times \frac{\pi^2}{4} \times \frac{1}{1+0} \; (5) + \int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x} dx$$

$$= \frac{-\pi^2}{8} + \int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x} dx$$

$$= \frac{-\pi^2}{8} + J. \quad (5)$$

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$$J = \int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x} dx = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - x}{\underbrace{1+\sin 2\left(\frac{\pi}{2} - x\right)}_{\sin 2x}} dx \quad (5)$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{1}{1+\sin 2x} dx - \underbrace{\int_0^{\frac{\pi}{2}} \frac{x}{1+\sin 2x} dx}_{J} \quad (5)$$

$$\therefore 2J = \frac{\pi}{2} \underbrace{\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin 2x} dx}_{1} \quad (5)$$

$$\therefore J = \frac{\pi}{4} \quad (5)$$

$$\therefore I = \frac{-\pi^2}{8} + \frac{\pi}{4} = \frac{\pi}{8}(2-\pi) \quad (5)$$

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---

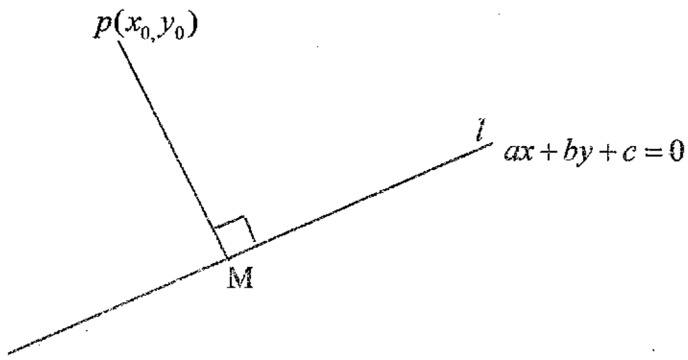
16. Let $P \equiv (x_0, y_0)$ and l be the straight line given by $ax + by + c = 0$. Show that the perpendicular distance from P to l is $\dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.

Let l_1 and l_2 be two straight lines given by $4x - 3y + 8 = 0$ and $3x - 4y + 13 = 0$, respectively. Show that l_1 and l_2 intersect at $A \equiv (1, 4)$.

Also, show that the parametic equations of the bisector of the acute angle between l_1 and l_2 can be written as $x = t$ and $y = t + 3$, where $t \in \mathbb{R}$.

Hence, show that the equation of any circle touching both straight lines l_1 and l_2, and lying in the region between l_1 and l_2 that contains the acute angle, is given by $(x-t)^2 + (y-t-3)^2 = \frac{1}{25}(t-1)^2$, where $t \in \mathbb{R}$ and $t \neq 1$.

From among the above circles, find the equations of the circles that intersect the circle centred at A of radius 1, orthogonally.



இங்கு $a^2 + b^2 \neq 0$

நேர்கோடு PM இன் சமன்பாடு $(y - y_0) = \frac{a}{b}(x - x_0)$ (5)

P இனூடாக செல்வதும் l இற்கு செங்குத்தானதுமான கோட்டிலுள்ள யாதாயினும் ஒரு புள்ளி

$(x_0 + at, y_0 + bt)$ for $t \in \mathbb{R}$. இனால் தரப்படும். (5)

M ஆனது l இல் உள்ளது; $a(x_0 + at) + b(y_0 + bt) + c = 0$ (5)

$\therefore t(a^2 + b^2) = -ax_0 + by_0 + c$

$\therefore \quad t = \dfrac{-(ax_0 + by_0 + c)}{a^2 + b^2}$ (5)

$\therefore$ தேவையான தூரம் $PM = \sqrt{a^2t^2 + b^2t^2}$ (5)

$= \sqrt{a^2 + b^2}\,|t|$

$= \dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$ (5)

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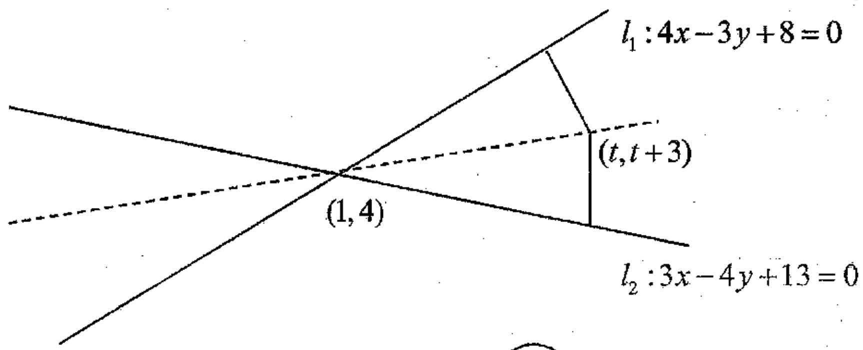

(5) (5)

A இன் ஆள்கூறுகளை l_1 and l_2 இல் பிரதியிட, l_1, and l_2 ஆனது $A = (1, 4)$ (5) இல் இடைவெட்டும்

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கோண இரு கூறாக்கிகள் $\dfrac{4x - 3y + 8}{5} = \pm \dfrac{3x - 4y + 13}{5}$ இனால் தரப்படும் (10)

The angle bisectors are $\underbrace{x + y - 5}_{m=-1} = 0$ and $x - y + 3 = 0.$ (5) (5)

Let θ be the acute angle between l_1 and $x_1 + y - 5 = 0$

Then, $\tan\theta = \left| \dfrac{\frac{4}{3} - (-1)}{1 + \frac{4}{3}(-1)} \right| = 7 > 1$ (5) (10) (5)

$\therefore$ கூர்ங்கோண இரு கூறாக்கி $x - y + 3 = 0$ (5)

கூர்ங்கோண இரு கூறாக்கி பரமனத்தில் கீழே தரப்பட்டுள்ளது.

Let $x=t$ for $t\in\mathbb{R}$. (5)

Then $y=x+3=t+3$. (5)

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தேவையான வட்டத்தின் மையம் கூர்ங்கோண இரு கூறாக்கியில் இருக்க வேண்டும் (5)

$\therefore$ மையம் $(t,t+3)$ for $t\in\mathbb{R}$ எனும் வடிவில் இருக்கும்

ஆரை $=\dfrac{|4t-3(t+3)+8|}{5}=\left|\dfrac{t-1}{5}\right|$ (5) (5)

$\therefore$ சமன்பாடு

$$(x-t)^2+(y-(t+3))^2=\tfrac{1}{25}(t-1)^2$$ (5)

That is $(x-t)^2+(y-t-3)^2=\tfrac{1}{25}(t-1)^2$, where $t\in\mathbb{R}$. (5)

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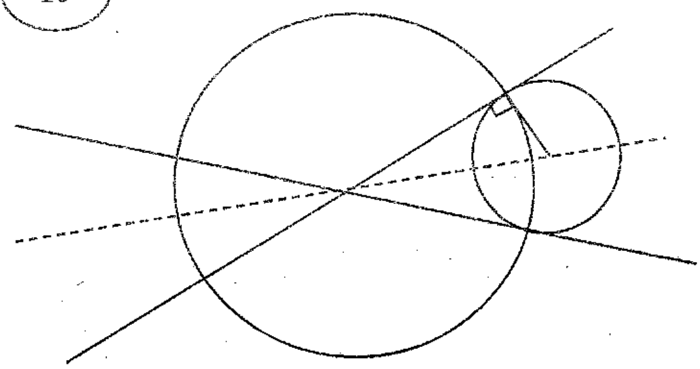
நிமிர்கோணத்தில் இடைவெட்டும் வட்டங்களுக்கு பைதகரசு தேற்றத்தை பிரயோகிக்க

(10)

$$(t-1)^2+(t+3-4)^2=1^2+\frac{1}{25}(t-1)^2$$



$\therefore\quad (t-1)^2=25$

$\Rightarrow t-1=5$ or $t-1=-5$

$\therefore\quad t=6$ or $t=-4$ (5) (5)

$\therefore$ Equation of circle that intersects S orthogonally an $(x-6)^2+(y-9)^2=1,\ (x+4)^2+(y-7)^2=1$ (5) (5)

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17. (*a*) Write down $\cos(A+B)$ in terms of $\cos A$, $\cos B$, $\sin A$ and $\sin B$, and obtain a similar expression for $\sin(A-B)$.

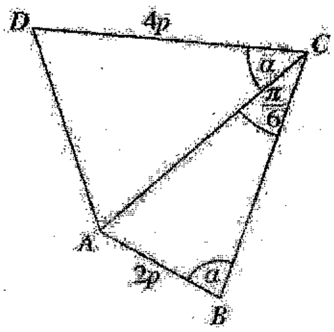
Let $k \in \mathbb{R}$ and $k \neq 1$. By separately considering the cases $k>1$ and $k<1$, express

$2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)$ in the form $R\cos(\theta+\alpha)$, where $R(>0)$ in terms of k, and $\alpha\,(0<\alpha<2\pi)$ are real constants to be determined.

Hence, solve $2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)=|k-1|$.

(*b*) In the quadrilateral $ABCD$ shown in the figure $AB=2p$, $CD=4p$, $A\hat{C}B=\frac{\pi}{6}$ and $A\hat{B}C=A\hat{C}D=\alpha$. Show that $AD^2=16p^2(\sin^2\alpha-\sin 2\alpha+1)$.

Hence, show that if $AD=4p$, then $\alpha=\tan^{-1}(2)$.



(*c*) Solve, $\tan^{-1}(\ln x^{\frac{2}{3}})+\tan^{-1}(\ln x)+\tan^{-1}(\ln x^2)=\frac{\pi}{2}$ for $x>1$.

(*a*) $\cos(A+B)=\cos A\cos B-\sin A\sin B$ (5)

$$\sin(A-B)=\cos\left(\frac{\pi}{2}-(A-B)\right) \quad (5)$$

$$=\cos\left((\frac{\pi}{2}-A)+B\right)$$

$$=\cos\left(\frac{\pi}{2}-A\right)\cos B-\sin\left(\frac{\pi}{2}-A\right)\sin B \quad (5)$$

$$=\sin A\cos B-\cos A\sin B \quad (5)$$

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$$2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)$$

$$=2k\left(\cos\theta\cos\frac{\pi}{3}-\sin\theta\sin\frac{\pi}{3}\right)+2\left(\sin\theta\cos\frac{\pi}{6}-\cos\theta\sin\frac{\pi}{6}\right) \quad (10)$$

$$=k\left(\cos\theta-\sqrt{3}\sin\theta\right)+\left(\sqrt{3}\sin\theta-\cos\theta\right) \quad (5)$$

$$=(k-1)\left(\cos\theta-\sqrt{3}\sin\theta\right)$$

$$=2(k-1)\left(\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right) \quad (5)$$

$$=2(k-1)\cos(\theta+\beta) \quad \text{where } \beta=\frac{\pi}{3} \quad (5)$$

when $k>1$ $\quad 2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)=2(k-1)\cos\left(\theta+\frac{\pi}{3}\right)$

where $R=2(k-1)$ and $\alpha=\frac{\pi}{3}.$ (5)

when $k<1$ $\quad 2k\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)=2(1-k)\cos\left(\pi+\theta+\frac{\pi}{3}\right)$

$$=2(1-k)\cos\left(\theta+\frac{4\pi}{3}\right)$$

where $R=2(k-1)$ and $\alpha=\frac{4\pi}{3}.$ (5)

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$2k\cos\left(\theta+\frac{\pi}{3}\right)+2\sin\left(\theta-\frac{\pi}{6}\right)=|k-1|$

when $k>1$

$2(k-1)\cos\left(\theta+\frac{\pi}{3}\right)=k-1$

$\therefore \cos\left(\theta+\frac{\pi}{3}\right)=\frac{1}{2}$ (5)

$\Rightarrow \theta+\frac{\pi}{3}=2n\pi\pm\frac{\pi}{3} \quad n\in\mathbb{Z}$

$\therefore \quad \theta=2n\pi-\frac{\pi}{3}\pm\frac{\pi}{3} \ n\in\mathbb{Z}.$ (5)

when $k<1$

$2(1-k)\cos\left(\theta+\frac{4\pi}{3}\right)=1-k$

$\therefore \quad \cos\left(\theta+\frac{4\pi}{3}\right)=\frac{1}{2}$ (5)

$\theta+\frac{4\pi}{3}=2n\pi\pm\frac{\pi}{3}n\in\mathbb{Z}.$

$\therefore \quad \theta=2n\pi-\frac{4\pi}{3}\pm\frac{\pi}{3}n\in\mathbb{Z}.$

(5)

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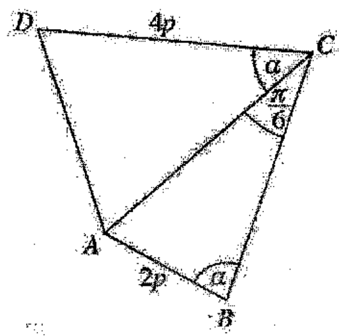
(*b*) முக்கோணம் ABC: இற்கு சைன் விதி (10)



$$\frac{b}{\sin\alpha}=\frac{2p}{\sin\frac{\pi}{6}}\Rightarrow b=4p\sin\alpha \quad (5)$$

Cosine Rule for the triangle ACD: (10)

$$AD^2=b^2+(4p)^2-2b(4p)\cos\alpha$$

$$=16p^2\sin^2\alpha+16p^2-2(4p)^2\sin\alpha\cos\alpha$$

$$=16p^2(\sin^2\alpha-\sin2\alpha+1) \quad (5)$$

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If $AD=4p$, the ADC is an isosceles triangle, we have

$\sin^2\alpha-\sin2\alpha+1=1$

$\sin\alpha(\sin\alpha-2\cos\alpha)=0$ (5)

Since $\sin\alpha\neq0$,

$\sin\alpha=2\cos\alpha$ (5)

$\frac{\sin\alpha}{\cos\alpha}=2$ $\qquad\cos\alpha\neq0$

$\therefore\quad\tan\alpha=2$

$\alpha=\tan^{-1}(2)$ (5)

15

(*c*)

For $x>1$

$$\underbrace{\tan^{-1}\left(\ln x^{2/3}\right)}_{\alpha}+\underbrace{\tan^{-1}\left(\ln x\right)}_{\beta}+\underbrace{\tan^{-1}\left(\ln x^{2}\right)}_{\theta}=\frac{\pi}{2}$$

$$\beta+\theta=\frac{\pi}{2}-\alpha \quad (5)$$

$$\tan(\beta+\theta)=\cos\alpha \quad (5)$$

$$\frac{\tan\beta+\tan\theta}{1-\tan\beta\tan\theta}=\frac{1}{\tan\alpha} \quad (5)$$

$$\therefore \quad \frac{\ln x+\ln x^2}{1-\ln x\ln x^2}=\frac{1}{\ln x^{\frac{2}{3}}} \quad (5)$$

$$\frac{\ln x^3}{1-2(\ln x)^2}=\frac{1}{\frac{2}{3}\ln x}$$

Taking $t=\ln x$

$$3x\frac{2}{3}t^2=1-2t^2 \quad (5)$$

$$4t^2=1$$

$$\ln x=t=\frac{1}{2} \qquad (\therefore t\neq\frac{-1}{2} \text{ as } t=\ln x \text{ and } x>1)$$

$$\therefore \quad x=e^{\frac{1}{2}} \quad (5)$$

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Verification

$$\tan^{-1}\left(\ln\left(e^{\frac{1}{2}}\right)^{\frac{2}{3}}\right)+\tan^{-1}\left(\ln e^{\frac{1}{2}}\right)+\tan^{-1}(\ln e)\doteq\frac{\pi}{2}.$$

$$\Leftrightarrow \qquad \underbrace{\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)}_{\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}}\doteq\frac{\pi}{4}.$$

$$=\frac{\frac{5}{6}}{\frac{5}{6}}$$

$$=1$$

අந்தரங்கமானது

இலங்கைப் பரீட்சைத் திணைக்களம்

க.பொ.த (உயர் தர)ப் பரீட்சை – 2022(2023)

10 - இணைந்த கணிதம் II

புள்ளியிடும் திட்டம்

இந்த விடைத்தாள் பரீட்சகர்களின் உபயோகத்திற்காகத் தயாரிக்கப்பட்டது. பிரதம பரீட்சகர்களின் கலந்துரையாடல் நடைபெறும் சந்தர்ப்பத்தில் பரிமாறிக்கொள்ளப்படும் கருத்துக்களுக்கேற்ப இதில் உள்ள சில விடயங்கள் மாற்றப்படலாம்.

இறுதித்திருத்தங்கள் உள்ளடக்கப்படவுள்ளன

க.பொ.த (உயர் தர)ப் பரீட்சை - 2022(2023)

10 - இணைந்த கணிதம் II

புள்ளி வழங்கும் திட்டம்

பகுதி II

பகுதி A	=	10×25	=	250
பகுதி B	=	05×150	=	750

மொத்தம் $= \dfrac{1000}{10}$

இறுதிப் புள்ளி = **100**

Common Techniques of Marking Answer Scripts.

It is compulsory to adhere to the following standard method in marking answer scripts and entering marks into the mark sheets.

1. Use a red color ball point pen for marking. (Only Chief/Additional Chief Examiner may use a mauve color pen.)
2. Note down Examiner's Code Number and initials on the front page of each answer script.
3. Write off any numerals written wrong with a clear single line and authenticate the alterations with Examiner's initials.
4. Write down marks of each subsection in a △ and write the final marks of each question as a rational number in a □ with the question number. Use the column assigned for Examiners to write down marks.

Example: **Question No. 03**

(i) .... √ △ $\frac{4}{5}$

(ii) .... √ △ $\frac{3}{5}$

(iii) .... √ △ $\frac{3}{5}$

(03) (i) $\frac{4}{5}$ + (ii) $\frac{3}{5}$ + (iii) $\frac{3}{5}$ = □ $\frac{10}{15}$

MCQ answer scripts: (Template)

1. Marking templets for G.C.E.(A/L) and GIT examination will be provided by the Department of Examinations itself. Marking examiners bear the responsibility of using correctly prepared and certified templates.
2. Then, check the answer scripts carefully. If there are more than one or no answers Marked to a certain question write off the options with a line. Sometimes candidates may have erased an option marked previously and selected another option. In such occasions, if the erasure is not clear write off those options too.
3. Place the template on the answer script correctly. Mark the right answers with a 'V' and the wrong answers with a 'X' against the options column. Write down the number of correct answers inside the cage given under each column. Then, add those numbers and write the number of correct answers in the relevant cage.

Structured essay type and assay type answer scripts:

1. Cross off any pages left blank by candidates. Underline wrong or unsuitable answers. Show areas where marks can be offered with check marks.

2. Use the right margin of the overland paper to write down the marks.
3. Write down the marks given for each question against the question number in the relevant cage on the front page in two digits. Selection of questions should be in accordance with the instructions given in the question paper. Mark all answers and transfer the marks to the front page, and write off answers with lower marks if extra questions have been answered against instructions.

4. Add the total carefully and write in the relevant cage on the front page. Turn pages of answer script and add all the marks given for all answers again. Check whether that total tallies with the total marks written on the front page.

Preparation of Mark Sheets.

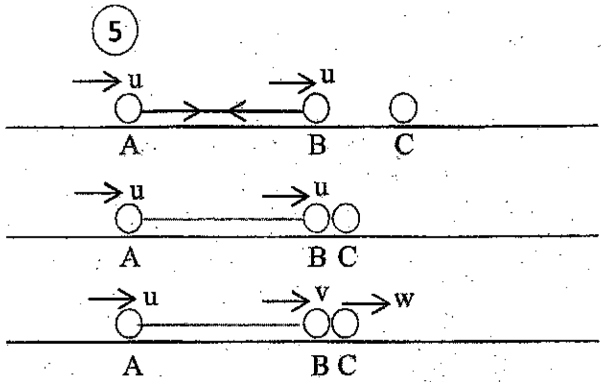
Except for the subjects with a single question paper, final marks of two papers will not be calculated within the evaluation board this time. Therefore, add separate mark sheets for each of the question paper. Write paper 01 marks in the paper 01 column of the mark sheet and write them in words too. Write paper II Marks in the paper II Column and wright the relevant details.

***

Part A

1. Three particles A, B and C, each of mass m, are placed in a straight line on a smooth horizontal table with A and B distance a apart, and connected by a light inextensible string of length a, as shown in the figure.

The particle B is given an impulse in the direction of $\overrightarrow{AB}$ such that its velocity just after the impulse is u. Show that the velocity of B just after it collides with C is $\frac{1}{2}(1-e)u$ in the direction of $\overrightarrow{AB}$, where e is the coefficient of restitution between B and C.
Also, find the time taken, after this collision, for A to collide with B.



$\underline{I} = \Delta(m\underline{v})$ for B and C : பிரயோகிக்க

$\rightarrow$ $0 = mv + mw - mu$ (5)

$\therefore v + w = u$ ------------- (1)

நியூட்டன் மீளமைவு விதியைப் பிரயோகிக்க:

$w - v = eu$ ------------- (2) (5)

$(1) - (2): 2v = u - eu$

$\therefore v = \frac{1}{2}(1-e)u$ (5)

எடுத்த நேரம் $= \dfrac{a}{u-v}$

$= \dfrac{2a}{(1+e)u}$ (5)

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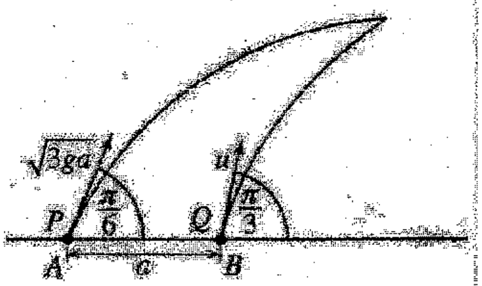
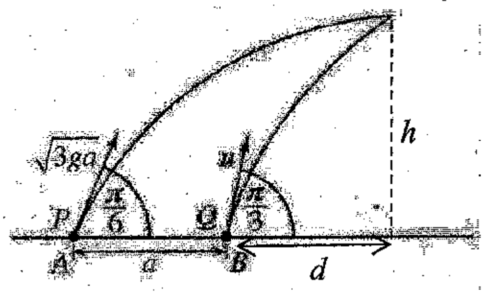
2. A and B are two points on a horizontal ground such that $AB = a$. Two particles P and Q are projected from the points A and B respectively, at the same instant and in the vertical plane that contains the line AB such that they collide with each other after a time T at a point in space. The initial velocities of P and Q are given in the figure.
Show that $u = \sqrt{ga}$ and find T in terms of a and g.





$s = ut + \frac{1}{2}at^2$:இனை பிரயோகிக்க

$$(P)\uparrow \quad h = \sqrt{3ga}\ .\ \frac{1}{2}\ T - \frac{1}{2}gT^2 \text{ ---------------(1)} \quad (5)$$

$$(Q)\uparrow \quad h = u\ .\ \frac{\sqrt{3}}{2}\ .\ T - \frac{1}{2}gT^2 \text{ ---------------(2)} \quad (5)$$

$$(1)-(2):\quad u\ \frac{\sqrt{3}}{2}\ T = \sqrt{3ga}\ .\ \frac{1}{2}\ T \quad (5)$$

$$\Rightarrow u = \sqrt{ga}$$

$$(P)\longrightarrow \quad a + d = \sqrt{3ga}\ \frac{\sqrt{3}}{2}\ T$$

(இரண்டுக்கும்) (5)

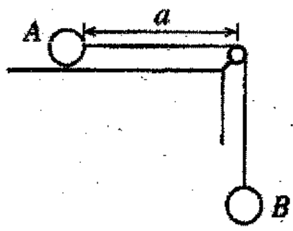
$$(Q)\longrightarrow \quad d = \sqrt{ag}\ .\ \frac{1}{2}\ .\ T$$

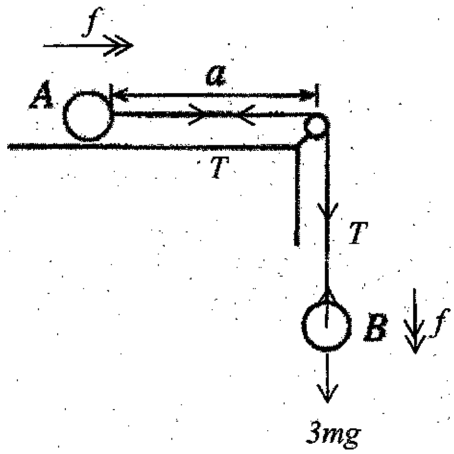
$$\therefore\ a + \frac{\sqrt{ag}}{2}\ T = 3\frac{\sqrt{ag}}{2}\ T$$

$$\Rightarrow\ a = 2\frac{\sqrt{ag}}{2}\ T$$

$$\Rightarrow\ T = \sqrt{\frac{a}{g}} \quad (5)$$

3. Two particles A and B of masses m and $3m$, respectively, are attached to the ends of a light inextensible string. The particle A is held at rest on a horizontal table with the string passing over a small smooth pulley fixed at the edge of the table. The particle B hangs vertically below the pulley. The system is released from rest with the particle A at a distance a from the pulley. In the subsequent motion, a constant frictional force of magnitude $\frac{1}{2}mg$ acts on A. Find the acceleration of A.
Also, find the speed of A at the instant when A reaches the pulley.





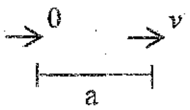
$\vec{F} = m\vec{a}$ இனை பிரயோகிக்க

B இற்கு : $\downarrow 3mg - T = 3mf$ ------------(1) (5)

A இற்கு : $\rightarrow T - \frac{1}{2}mg = mf$ ------------(2) (5)

(1) − (2) : $\frac{5}{2}mg = 4mf$

$f = \frac{5}{8}g$ (5)



$v^2 = u^2 + 2as$: இனை பிரயோகிக்க

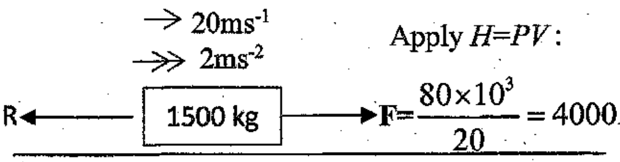
$v^2 = 2fa.$ (5)

$\therefore v^2 = 2 \times \frac{\sqrt{5ag}}{8}.$

$\therefore v = \frac{\sqrt{5ag}}{2}.$ (5)

25

4. A car of mass 1500 kg working with a constant power 80 kW moves on a horizontal road against a constant resistance. The acceleration of the car is $2\ m\ s^{-2}$, when it moves with the speed $20\ m\ s^{-1}$. Obtain equations sufficient to determine the acceleration of the car when it moves upward on a road of inclination $\sin^{-1}\left(\frac{2}{3}\right)$ to the horizontal with speed $8\ m\ s^{-1}$ working with the same constant power against the same constant resistance.



Apply $H=PV$:

$F=\dfrac{80\times10^3}{20}=4000N$ (5)

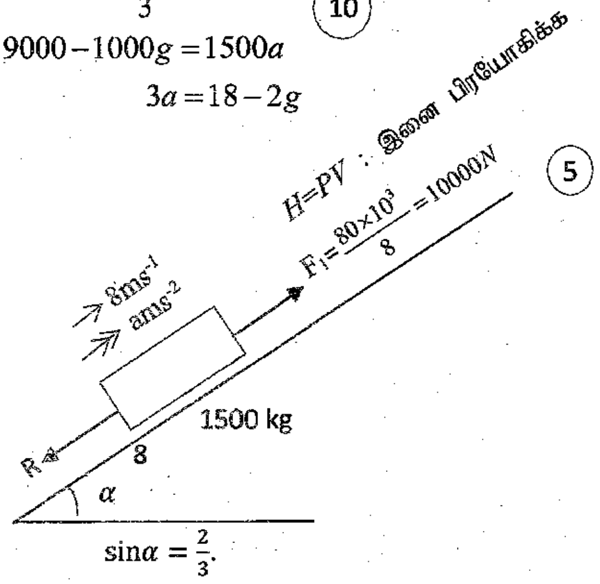
$\vec{F}=m\vec{a}$: இனை பிரயோகிக்க $\longrightarrow$ $4000-R=1500\times2$ (5)

$\therefore R=1000N$

$10000-1000-1500\times\dfrac{2}{3}g=1500a$ (10)

$9000-1000g=1500a$

$3a=18-2g$



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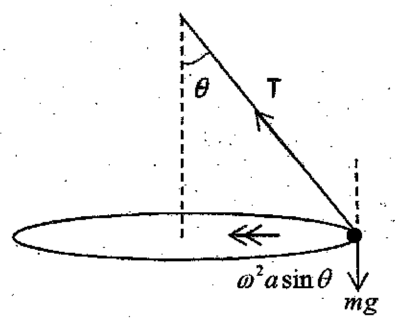
5. One end of a light inextensible string of length a is attached to a fixed point and the other end to a particle of mass m. The particle moves in a horizontal circle with constant angular speed ω. The string makes an angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ with the downward vertical. Show that $\omega>\sqrt{\frac{g}{a}}$.

$\vec{F}=m\vec{a}$ இனை பிரயோகிக்க

$\uparrow \; T\cos\theta = mg$ ---------------(1) (5)



$\leftarrow T\sin\theta = m\omega^2 a\sin\theta$ ---------------(2) (5)

$\therefore T = m\omega^2 a$

(1), (2) : $\Rightarrow \cos\theta = \frac{g}{\omega^2 a}$ (5)

$0<\theta<\frac{\pi}{2}$, we have $\cos\theta<1$. (5)

$\therefore \frac{g}{\omega^2 a}<1.$

$\therefore \omega > \sqrt{\frac{g}{a}}$ (5)

25

6. **In the usual notation, the position vectors of two points A and B with respect to a fixed origin O are $3\mathbf{i}+2\mathbf{j}$ and $2\mathbf{i}+4\mathbf{j}$, respectively. Show that O, A and B are non-collinear.**
Let C be the point such that $\overrightarrow{BC}=\lambda\overrightarrow{OA}$, where $\lambda\in\mathbb{R}$. Find $\overrightarrow{OC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and λ.
Show that if $B\hat{O}C=\frac{\pi}{2}$, then $\lambda=-\frac{10}{7}$.

$3:2\neq 2:4$, ஆதலால் O, A , B என்பன ஒரே நேர்கோட்டில் இல்லாத புள்ளிகளாகும். (5)

$\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}$ (5)

$=2\underline{i}+4\underline{j}+\lambda(3\underline{i}+2\underline{j})$.

$\therefore \overrightarrow{OC}=(2+3\lambda)\underline{i}+(4+2\lambda)\underline{j}$. (5)

since $B\hat{O}C=\frac{\pi}{2}$, we have $\overrightarrow{OB}\cdot\overrightarrow{OC}=0$.

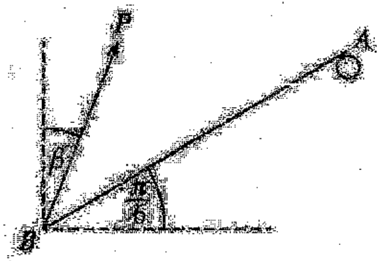
$\therefore (2i+4j)\cdot((2+3\lambda)\underline{i}+(4+2\lambda)\underline{j})=0$. (5)

$\therefore\ 4+6\lambda+16+8\lambda=0$.

$\therefore\ \lambda=\frac{-10}{7}$. (5)

25

7. A uniform rod AB is kept in equilibrium with its upper end A resting on a smooth peg by applying a force P to its lower end B at an angle β to the vertical, as shown in the figure. The rod makes an angle $\frac{\pi}{6}$ with the horizontal. Show that $\tan\beta = \frac{\sqrt{3}}{5}$.



$\triangle BMN;\ BM = a\cos\frac{\pi}{6} = a\frac{\sqrt{3}}{2}$ (5)

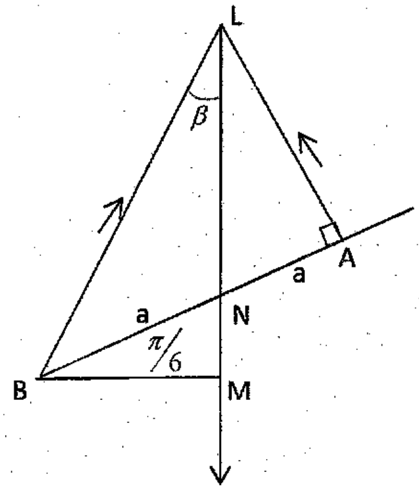
$MN = a\sin\frac{\pi}{6} = \frac{a}{2}$ (5)

$\triangle ALN;\ LN = \frac{a}{\cos\frac{\pi}{3}} = 2a$ (5)

$\therefore LM = 2a + \frac{a}{2} = \frac{5a}{2}.$ (5)

$\triangle BLM;\ \tan\beta = \frac{BM}{LM} = \frac{a\frac{\sqrt{3}}{2}}{5\frac{a}{2}} = \frac{\sqrt{3}}{5}.$ (5)

$\tan\beta = \frac{\sqrt{3}}{5}.$



வேறுமுறை:

$B \circlearrowright\ W\,a\cos\frac{\pi}{6} = R.(2a) \Rightarrow R = \frac{\sqrt{3}W}{4}.$ (5)

$\uparrow\quad P\cos\beta + R\cos\frac{\pi}{6} = W$ (5)

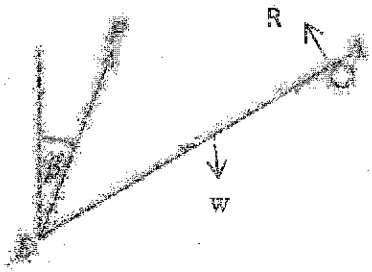
$P\cos\beta = W - \frac{\sqrt{3}}{4}\ .\ \frac{\sqrt{3}W}{2} = \frac{5W}{8}$ (5)

$\rightarrow\quad P\sin\beta = R\sin\frac{\pi}{6}$ (5)

$= \frac{\sqrt{3}W}{4}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}W}{8}$

$\therefore \tan\beta = \frac{\sqrt{3}W}{8} \div \frac{5W}{8} = \frac{\sqrt{3}}{5}$ (5)



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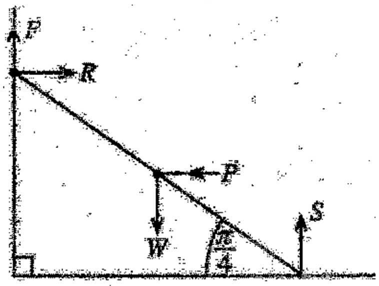
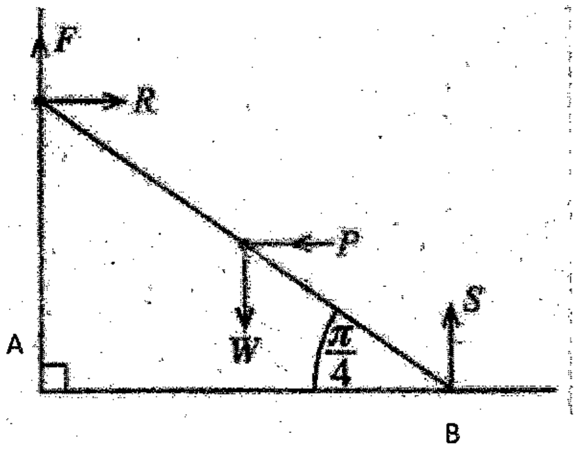
8. A uniform ladder of weight W and length $2a$ is kept in equilibrium against a rough vertical wall with its lower end on a smooth horizontal ground, by a horizontal force of magnitude P applied at the mid-point of the ladder as shown in the figure. The ladder makes an angle $\frac{\pi}{4}$ with the ground. The coefficient of friction between the ladder and the wall is $\frac{1}{6}$. Show that, $\frac{3W}{4} \le P \le \frac{3W}{2}$.





ஏணியின் சமநிலைக்கு:

$\uparrow \quad F + S = W$ (5)

$\leftarrow \quad P = R$ (5)

$A\circlearrowleft \quad W\,a\cos\frac{\pi}{4} + P \cdot a \cdot \sin\frac{\pi}{4} - S \cdot 2a\cos\frac{\pi}{4} = 0$ (5)

$\therefore \quad S = \frac{W+P}{2},$

and $= \frac{W-P}{2}.$

Now, $\frac{1}{6} \ge \frac{|F|}{R}$

$\Rightarrow -\frac{1}{6} \le \frac{W-P}{2P} \le \frac{1}{6}$

$\Rightarrow -P \le 3(W-P) \le P$

$\Rightarrow \frac{3W}{4} \le P \le \frac{3W}{2}.$ (10)

25

9. Let A and B be two events of a sample space Ω. It is given that $P(A)=\frac{2}{7}$, $P(A\cup B)=\frac{11}{14}$ and $P(A'\cup B')=\frac{4}{5}$. Find $P(B)$ and show that A and B are independent events.

$$P(A\cup B)=P(A)+P(B)-P(A\cap B) \quad (5)$$

$$\Rightarrow \frac{11}{14}=\frac{2}{7}+P(B)-\frac{1}{5}.$$

$$\therefore P(B)=\frac{7}{10}. \quad (5)$$

$$P(A\cap B)=1-P(A'\cup B')=\frac{1}{5} \quad (5) \quad (5)$$

$$P(A)P(B)=\frac{2}{7}\times\frac{7}{10}=\frac{1}{5}=P(A\cap B). \quad (5)$$

$\therefore A$ and B are independent.

25

10. The mean and the standard deviation of marks obtained by 100 students for an examination are 60 and 20, respectively. Find the z-score of a student who obtained 56 marks for this examination. It was later found that this mark of 56 has been entered erroneously and it should have been 65 instead. Find the correct value of the mean of the marks obtained for this examination.

$$z=\frac{56-60}{20}=\frac{-4}{20}=\frac{-1}{5}=-0.2 \quad (5) \quad (5)$$

$$60=\mu_{old}=\frac{\sum_{i=1}^{100}x_i}{100} \Rightarrow \left(\sum_{i=1}^{100}x_i\right)_{old}=6000 \quad (5)$$

$$\therefore \mu_{correct}=\frac{\left(\sum_{i=1}^{100}x_i\right)_{correct}}{100}=\frac{6000-56+65}{100}=\frac{6009}{100}=60.09 \quad (5) \quad (5)$$

25

Part B

※ Answer five questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (a) A car P that begins its journey from rest from a point O on a straight horizontal road travels with a constant acceleration $2f$ m s^{-2} up to a point A on that road, where $OA = a$ m. It maintains the velocity attained at A throughout its remaining journey. At the instant when car P reaches the point A, another car Q begins its journey, along the same road in the same direction, from rest at the point O and moves with a constant acceleration f m s^{-2}. Sketch the velocity-time graphs for the motion of P and Q in the same diagram.

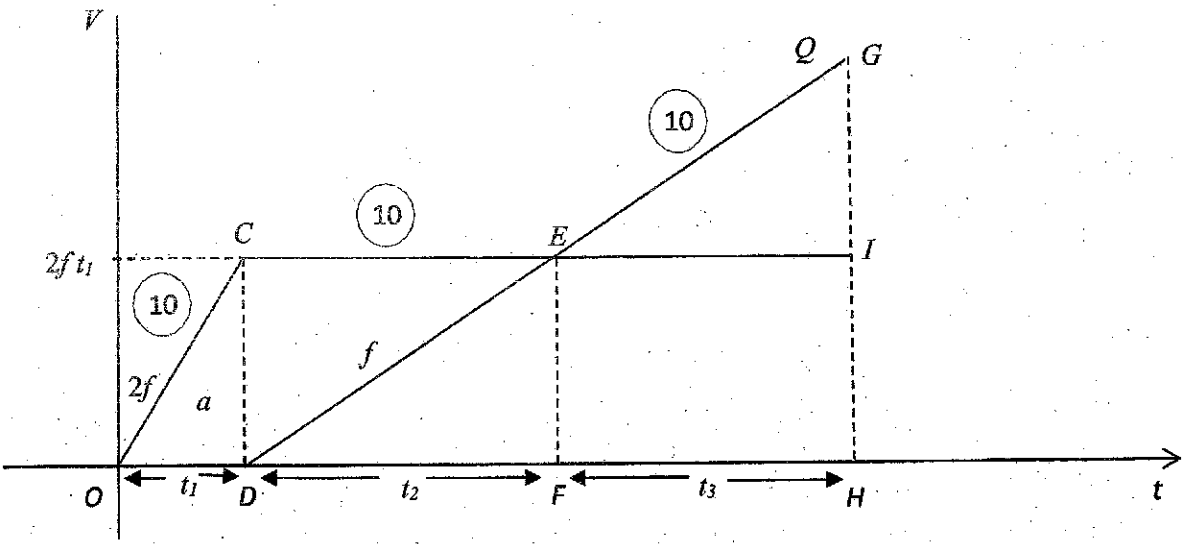
Hence, show that the time taken by Q to the instant when the velocities of P and Q are equal is $2\sqrt{\frac{a}{f}}$ s.

Now, let $a = 50$ and $f = 2$, and let B be the point on the road at which the car Q passes the car P. Show that $AB = 50\left(5+2\sqrt{6}\right)$ m.

(b) A ship P is sailing due South with a uniform speed 60 m s^{-1} relative to earth and a ship Q is sailing due East with a uniform speed $30\sqrt{3}$ m s^{-1} relative to earth. A third ship R appears to move in the direction 30° North of East when it is observed from P and ship R appears to move due South when it is observed from Q. Show that the ship R moves in the direction 30° South of East with a speed 60 m s^{-1} relative to earth.

Suppose that initially the ship R is located 24 km away from P in a direction 60° South of West and 6 km away from Q in due West. Show that the distance between Q and R is 12 km, when P and R are the shortest distance apart.

(a)



30

Δ OCD இலிருந்து :

$\frac{1}{2}(t_1)(2f\, t_1) = a$ (5)

$\Rightarrow \quad t_1^2 = \frac{a}{f}$

$\therefore \quad t_1 = \sqrt{\frac{a}{f}} \quad as \ t_1 > 0.$ (5)

Δ DEF இலிருந்து :

$f = \frac{2f\, t_1}{t_2}.$ (5)

$\therefore t_2 = 2t_1.$

$= 2\sqrt{\frac{a}{f}}$ (5)

20

Let $a = 50$ and $f = 2.$

Then $t_1 = \sqrt{\frac{50}{2}} = 5$ and $t_2 = 10.$ (5)

Q ஆனது P இனை B இல் சந்திப்பதால், $OCED$ இன் பரப்பு = EGI இன் பரப்பு.

$\therefore \frac{1}{2}(5+10)(2 \cdot 2 \cdot 5) = \frac{1}{2} + 3 \cdot 2t_3$ (5)

$t_3^2 = 150$

$t_3 = \sqrt{150} = 5\sqrt{6}.$ (5)

15

$AB = \frac{1}{2}(t_2 + t_3)(2f\, t_1 + f\, t_3)$ (5)

$= \frac{1}{2}\left(10 + 5\sqrt{6}\right)\left(5 \times 2 + 5\sqrt{6}\right) \cdot (2) = 50\left(5 + 5\sqrt{6}\right)$ (5)

10

(b)

$$\underline{V}(P,E) = \downarrow 60$$
$$\underline{V}(Q,E) = \rightarrow 30\sqrt{3}$$
$$\underline{V}(R,P) = \nearrow 30^0$$
$$\underline{V}(R,Q) = \downarrow$$

(10)

$$\underline{V}(R,E) = \underline{V}(R,P) + \underline{V}(P,E)$$
$$= \underline{V}(P,E) + \underline{V}(R,P)$$
$$= \underline{V}(R,P) + \underline{V}(P,E)$$
$$= \overrightarrow{AB} + \overrightarrow{AC}$$

ΔABC (15)

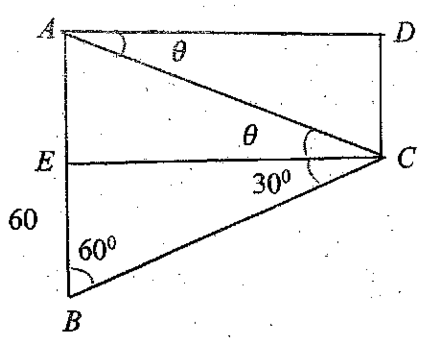
$$= \overrightarrow{AC}.$$

அத்துடன், $\underline{V}(R,E) = \underline{V}(R,Q) + \underline{V}(Q,E)$

$$= \underline{V}(Q,E) + \underline{V}(R,Q)$$
$$= \overrightarrow{AD} + \overrightarrow{DC}$$

ΔADC (15)

$$= \overrightarrow{AC}$$



$BE = 30\sqrt{3} \cdot \frac{1}{\sqrt{3}}$

$= 30.$

$\therefore AE = 30.$

Also, $CE = 30\sqrt{3}.$

$\tan\theta = \frac{AE}{CE} = \frac{1}{\sqrt{3}}$ (5)

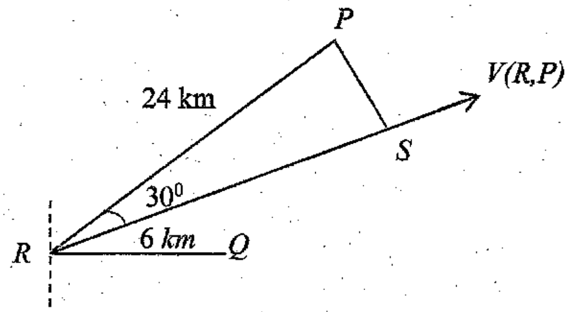
$\therefore \theta = 30^0$ (5)

Now $V^2 = \left(30\sqrt{3}\right)^2 + 30^2$ (5)

$V^2 = 30^2(4)$

$\therefore V = 60\text{ms}^{-1}$ (5)

60



$RS = 24000\sqrt{\frac{3}{2}}$

$= 12000\sqrt{3}$

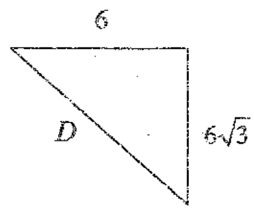
$t = \frac{12000\sqrt{3}}{60}$

$= 200\sqrt{3}$ S (5)

Let $d = 30 \times 200\sqrt{3} = 6000\sqrt{3}$

$= 6\sqrt{3}km$ (5)

$\therefore$ தேவையான தூரம் D km ஆனது:



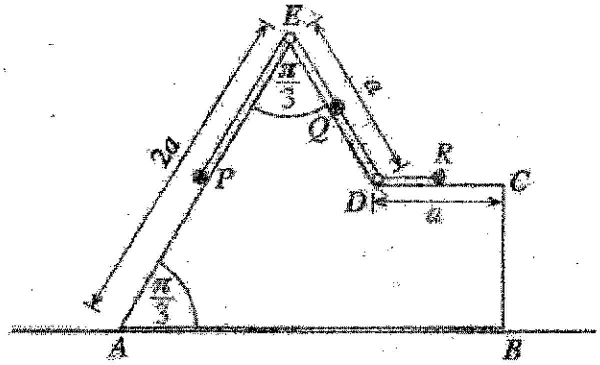
$D^2 = 6^2 + 6^2(3)$

$= 6^2(4)$

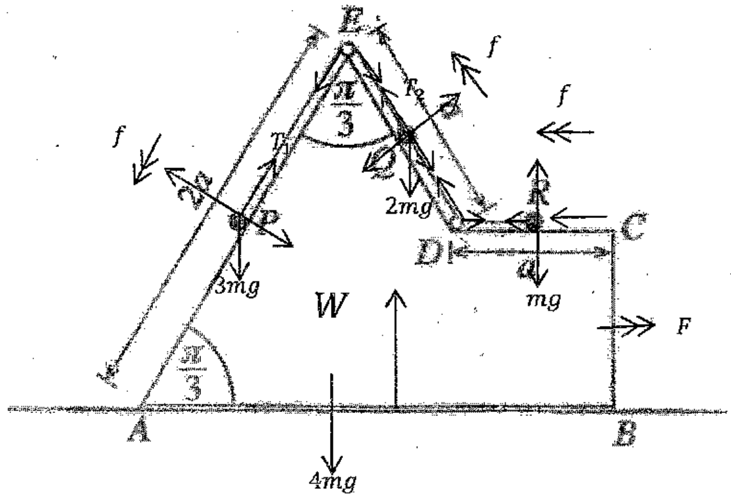
$\therefore D = 12$ km. (5)

15

12.(a) The vertical cross-section $ABCDE$ through the centre of gravity of a smooth uniform block of mass $4m$ is shown in the figure. The face containing AB is placed on a smooth horizontal floor. Also, AE and ED are the lines of greatest slope of the faces containing them. $AE = 2a$, $ED = a$, $DC = a$ and $E\hat{A}B = A\hat{E}D = \frac{\pi}{3}$. Three particles P, Q and R of masses $3m$, $2m$ and m, respectively, are placed at the mid-points of AE, ED and DC. The particles P and Q are attached to the ends of a light inextensible string passing over a smooth light small pulley fixed to the block at E, and the particles Q and R are attached to the ends of another light inextensible string passing through a smooth light small ring fixed to the block at D. Strings are taut in the position shown in the diagram and the system is released from rest from this position. Obtain equations sufficient to determine the time taken for the particle Q to reach E.



(a)



(10) விசைகளுக்கு

$\vec{V}(W,E) = \rightarrow F$

$\vec{V}(P,W) = \swarrow f$. எனின்

$\vec{V}(Q,W) = \nwarrow f$ (5)

$\vec{V}(R,W) = \leftarrow f$ (5)

$\underset{\sim}{F} = m\underset{\sim}{a}$:இனை பிரயோகிக்க

$P: \swarrow 3mg\cos\frac{\pi}{6} - T_1 = 3m(f - F\cos\frac{\pi}{3})$ (15)

$Q: \nwarrow T_1 - T_2 - 2mg\cos\frac{\pi}{6} = 2m(f - F\cos\frac{\pi}{3})$ (15)

$R: \leftarrow T_2 = m(f - F)$ (10)

தொகுதிக்கு:

$\rightarrow$

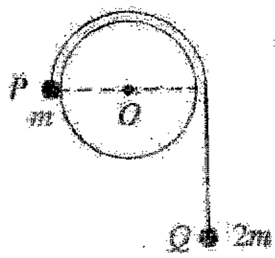
$$0 = 4mF + m(F - f) + 2m(F - f\cos\frac{\pi}{3}) + 3m(F - f\cos\frac{\pi}{3}) \quad (20)$$

$$Q \nwarrow s = ut + \frac{1}{2}ft^2$$

$$\Rightarrow \frac{a}{2} = \frac{1}{2}ft^2. \quad (10)$$

90

(b) A cylinder of radius a is fixed with its axis horizontal and the adjoining figure shows a vertical cross-section of the cylinder perpendicular to its axis. Two particles P and Q of masses m and $2m$, respectively connected by a light inextensible string are held with the string taut and OP horizontal in the position as shown in the figure and released from rest. Assuming that the particle Q moves vertically downwards, show that the speed v of the particle P when $\overrightarrow{OP}$ has turned through an angle θ $(0 \le \theta \le \frac{\pi}{6})$ is given by $v^2 = \frac{2ga}{3}(2\theta - \sin\theta)$.



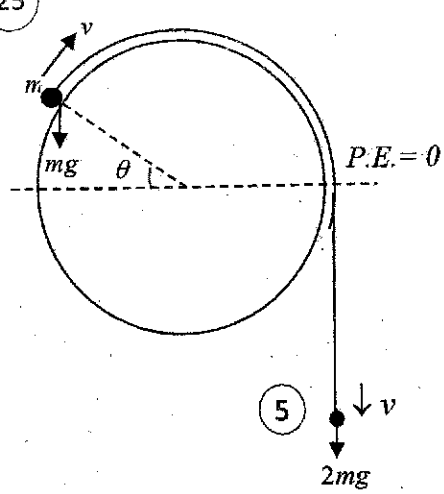
The string is cut when $\theta = \frac{\pi}{6}$ and it is given that the particle P moving on the cylinder comes to instantaneous rest before it reaches the highest point of the cylinder. In the subsequent motion, find the speed of P when it is at a distance a vertically below its initial position.

(b) சக்திக் காப்பின் படி

$$\frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + mga\sin\theta - 2ma\theta g = 0. \quad (25)$$

$$\Rightarrow 3v^2 = 2ag(2\theta - \sin\theta).$$

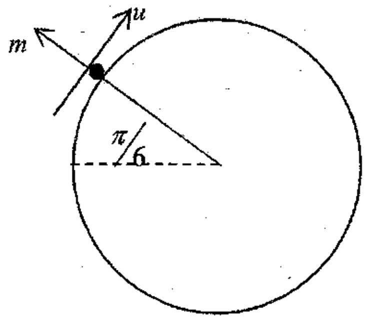
$$\Rightarrow v^2 = \frac{2ag}{3}(2\theta - \sin\theta). \quad (5)$$



35

$v = u$ when $\theta = \frac{\pi}{6}$ is given by $u^2 = \frac{2ag}{3}\left(\frac{\pi}{3} - \frac{1}{2}\right)$. (10)

$= \frac{ag}{9}(2\pi - 3).$



சக்திக் காப்பின் படி

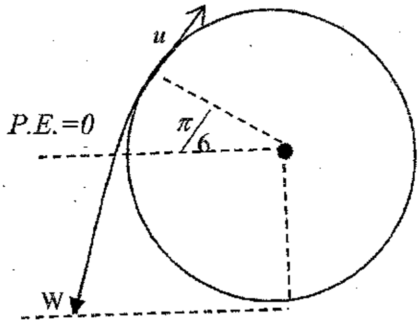
$$\frac{1}{2}mw^2 - mga = mg\frac{a}{2} + \frac{1}{2}mu^2$$ (10)

$$\frac{1}{2}mw^2 = \frac{3mga}{2} + \frac{1}{2}m\frac{ag}{9}(2\pi - 3)$$

$$\frac{1}{2}mw^2 = \frac{1}{2}mag\left[3 - \frac{1}{3} + \frac{2\pi}{9}\right]$$

$$w^2 = ag\left[\frac{8}{3} + \frac{2\pi}{9}\right] = \frac{ag}{9}[24 + 2\pi]$$

$$w = \frac{\sqrt{2ga(\pi + 12)}}{3}.$$ (5)



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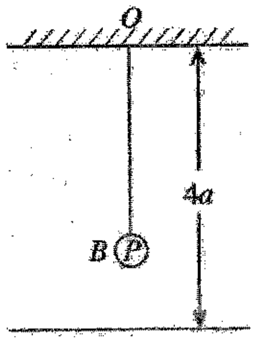
13. One end of a light elastic string of natural length $2a$ and modulus of elasticity $2mg$ is attached to a fixed point O which is at a distance of $4a$ above a smooth horizontal floor, and the other end to a particle P of mass m. The particle P hangs in equilibrium at B. Show that the extension of the string is a.



Now, the particle P is given an impulse of mv vertically downwards.

Show that the equation of motion of P is $\ddot{x}+\omega^2 x=0$ where $\omega=\sqrt{\dfrac{g}{a}}$ and $BP=x$.

Using the formula $\dot{x}^2=\omega^2(c^2-x^2)$, where c is the amplitude, show that if $v>\sqrt{ag}$, P hits the floor.

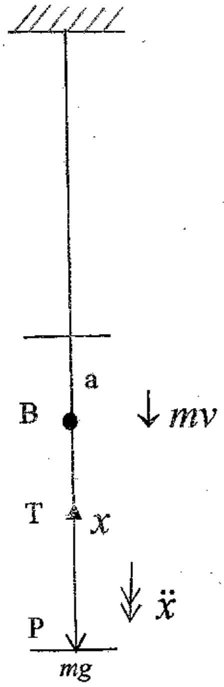
Now, suppose that $v=3\sqrt{ag}$.

Find the velocity with which P hits the floor.

The coefficient of restitution between P and the floor is e. If $e<\dfrac{1}{\sqrt{2}}$, show that the particle P will not reach O.

If it is given that $e=\dfrac{1}{2}$, find the velocity of P when the string becomes slack for the first time.

Find the total time taken by P to come to instantaneous rest for the first time, from the instant it was given the impulse at B.



நாப்பத்தானத்தில்,

$$2mg\cdot\frac{x}{2a}=mg. \quad (5)$$

$$\therefore\ x=a. \quad (5)$$

$\therefore$ இழையின் நீட்சி a ஆகும்.

10

$\underline{F}=m\underline{a}$:இனை பிரயோகிக்க

$$\downarrow m\ddot{x}=mg-2mg\frac{(a+x)}{2a} \quad (15)$$

$$\ddot{x}=-\frac{g}{a}x \quad (5)$$

$$\therefore\ \ddot{x}+\omega^2x=0,\ where\ \ \omega=\sqrt{\frac{g}{a}}.$$

20

$\dot{x} = v$ *when* $x = 0$

$\therefore v^2 = \omega^2 \left(c^2 - 0\right)$ (5)

$\therefore v = c\omega$

$\therefore c = \dfrac{v}{\omega}.$ (5)

if $v > \sqrt{ag}$, then $c > \sqrt{ag} \cdot \sqrt{\dfrac{a}{g}} = a$ (10)

and hence the particle hits the floor. **20**

$\dot{x} = u,$ where $x = a.$

Then

$u^2 = \dfrac{g}{a}\left(9a^2 - a^2\right) = 8ag,$ since $c = \dfrac{v}{w} = 3a.$ (5) (10)

$\therefore u = \sqrt{8ag}.$ (5) **20**

P ஆனது தரையை அடித்த உடன் வேகம் floor $= eu \uparrow$: (5)

$\therefore \dot{x} = eu,$ when $x = a.$

எ. இ. இ. யின் மையம் பற்றிய சமச்சீரின்படி, $\dot{x} = eu,$ when $x = -a.$ (15)

P இன் புவியீர்ப்பின் கீழான இயக்கத்திற்கு,

$v^2 = u^2 + 2as$: இனை பிரயோகிக்க

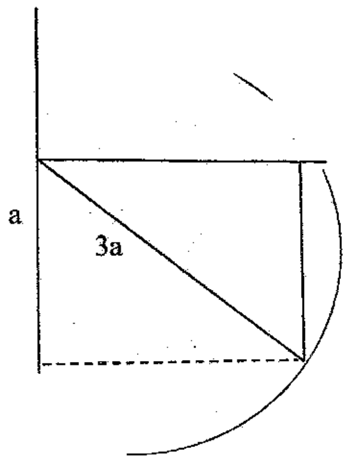
$\uparrow o = v_1^2 - 2gs$ (5)

$\therefore s = \dfrac{8e^2ag}{2g} = 4e^2a$ (5)

If $e < \dfrac{1}{\sqrt{2}}$, then $s < 2a$ at P will not reach O. (10) **40**

$e=\frac{1}{2}$, ஆகும் போது $v_1=\sqrt{8e^2ag}=\sqrt{2ag}$ (10)

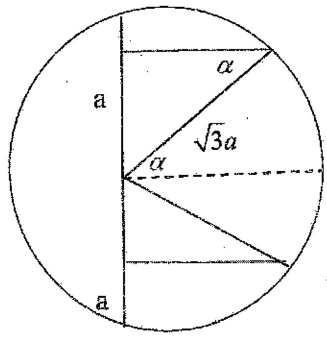
10



தரையை அடிக்க எடுத்த நேரம், $T_1=\dfrac{\sin^{-1}\left(\frac{1}{3}\right)}{\sqrt{\frac{g}{a}}}$ (10)

$=\sqrt{\frac{a}{g}}\sin^{-1}\left(\frac{1}{3}\right)$

Let $e=\frac{1}{2}$. Then $C_1=\sqrt{3}a$.



அதன் பின்னர் இயற்கை நீளத்தை அடைய எடுத்த நேரம்

Length $T_2=\dfrac{2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{\frac{g}{a}}}$. (10)

இந்த சுயாதீன இயக்கத்தின்போது: $\uparrow$ $V=u+at$.

$\therefore$ Time in free flight $T_3=\dfrac{\sqrt{2ag}}{g}=\sqrt{\dfrac{2a}{g}}$ (5)

எடுத்த மொத்த நேரம் $T_1+T_2+T_3=\sqrt{\frac{a}{g}}\left(\sin^{-1}\left(\frac{1}{3}\right)+2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right)$. (5)

30

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14. (*a*) Let the position vectors of four points A, B, C and D be $\mathbf{a}$, $\mathbf{b}$, $3\mathbf{a}$ and $4\mathbf{b}$, respectively with respect to a fixed origin O, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. E is the point of intersection of AD and BC. Using the triangle law of addition for the triangle OAE, show that $\overrightarrow{OE} = \mathbf{a} + \lambda(4\mathbf{b} - \mathbf{a})$ for $\lambda \in \mathbb{R}$.

Similarly, show also that $\overrightarrow{OE} = \mathbf{b} + \mu(3\mathbf{a} - \mathbf{b})$ for $\mu \in \mathbb{R}$.

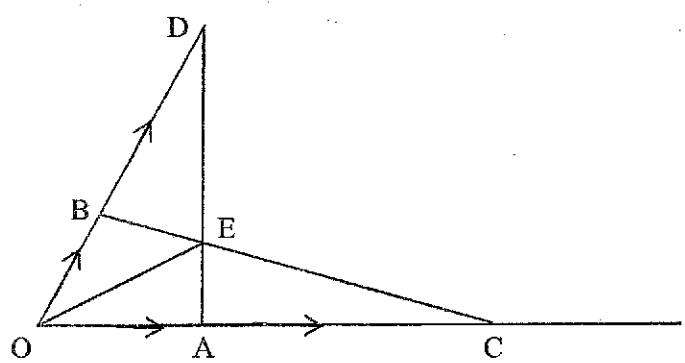
Hence, show that $\overrightarrow{OE} = \frac{1}{11}(9\mathbf{a} + 8\mathbf{b})$.

(*b*) Three forces $\alpha\mathbf{i} + 2\mathbf{j}$, $-3\mathbf{i} + \beta\mathbf{j}$ and $\mathbf{i} + 5\mathbf{j}$ act through the points with position vectors $\mathbf{i} + \mathbf{j}$, $3\mathbf{i} + \mathbf{j}$ and $2\mathbf{i} + 2\mathbf{j}$, respectively, where $\alpha, \beta \in \mathbb{R}$. It is given that this system of forces is equivalent to a couple. Find the values of α and β, and the moment of this couple.

Now, a new force $3\gamma\mathbf{i} + 4\gamma\mathbf{j}$ acting through the origin O is added to the above system of forces, where $\gamma > 0$. Show that the new system consisting of 4 forces is equivalent to a resultant force and, find its magnitude, direction and the equation of its line of action.

Next, it is given that when a force $p\mathbf{i} + q\mathbf{j}$ acting through the point with position vector $2\mathbf{i} + 3\mathbf{j}$ is added, the system consisting of 5 forces is in equilibrium. Find the values of γ, p and q.

(*a*)



OAE எனும் முக்கோணியிலிருந்து,

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$$
$$= \underline{a} + \lambda\overrightarrow{AD} \quad (5)$$
$$= \underline{a} + \lambda\left(\overrightarrow{AO} + \overrightarrow{OD}\right) \quad (5)$$
$$= \underline{a} + \lambda\left(4\underline{b} - \underline{a}\right) \quad (5)$$

OBE எனும் முக்கோணியிலிருந்து,

$$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$$
$$= \underline{b} + \mu\overrightarrow{BC} \quad (5)$$
$$= \underline{b} + \mu\left(\overrightarrow{BO} + \overrightarrow{OC}\right) \quad (5)$$
$$= \underline{b} + \mu\left(3\underline{a} - \underline{b}\right) \quad (5)$$

30

$\therefore\ \underline{a}+\lambda(4\underline{b}-\underline{a})=\underline{b}+\mu(3\underline{a}-\underline{b})$ (5)

$(1-\lambda)\underline{a}+4\lambda\underline{b}=3\mu\underline{a}+(1-\mu)\underline{b}$ (5)

$\Rightarrow 1-\lambda=3\mu$ and $1-\mu=4\lambda$ (5)

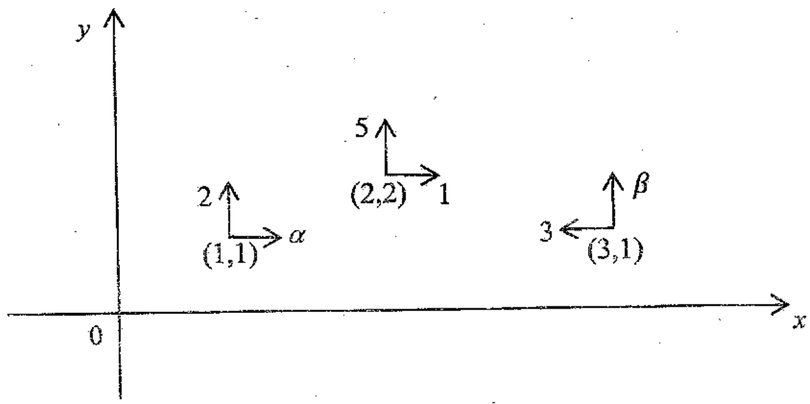
$\therefore\ \lambda=\dfrac{2}{11}$ (5)

$\therefore \overrightarrow{OE}=a+\dfrac{2}{11}(4\underline{b}-\underline{a})$ (5)

$=\dfrac{1}{11}(9\underline{a}+8\underline{b}).$ (5)

60

(*b*)



தொகுதி ஒரு இணைக்கு சர்வசமமானது ஆதலால்,

$\rightarrow X=0,\quad \uparrow Y=0$ and $G\neq 0.$

$X=\alpha-3+1=0.$ (5)

$\Rightarrow \alpha=2$ (5)

$Y=2+\beta+5=0.$ (5)

$\Rightarrow \beta=-7$ (5)

20

$O\circlearrowright G=2(1)-2(1)+3(1)-7(3)+5(2)-1(2)$ (5)

$=3-21+10-2$

$=13-23$

$=-10.$ (5)

10

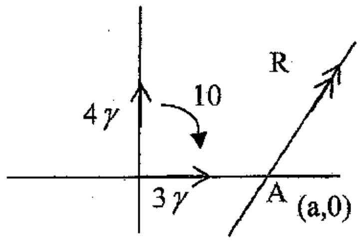
$R^2 = 9\gamma^2 + 16\gamma^2$ (5)

$= 25\gamma^2$

$\therefore R = 5\gamma.$ (5)

$\tan\theta = \frac{4\gamma}{3\gamma} = \frac{4}{3}$ (5)

$\therefore \theta = \tan^{-1}\left(\frac{4}{3}\right)$ (5)

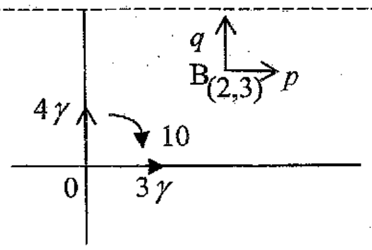


A↻ $4\gamma a = 10$

$\therefore a = \frac{5}{2\gamma}$ (5)

தாக்கக்கோட்டின் சமன்பாடு $4x - 3y - \frac{10}{\gamma} = 0.$ (5)

30



$\rightarrow p + 3\gamma = 0$ (5) $\uparrow q + 4\gamma = 0$ (5)

$\therefore p = -3\gamma$ $\therefore q = -4\gamma$

B↻ $(3\gamma \times 3) - (4\gamma \times 2) - 10 = 0$ (5)

$\therefore \gamma = 10.$ (5)

$\therefore p = -30$ (5) and $q = -40$ (5)

30

வேறுமுறை:

O↻ $q(2) - 3p - 4r(x) = 0$ (5)

$2q - 3p - 4r\left(\frac{5}{2r}\right) = 0$ (5)

$2q - 3p - 10 = 0$

O↻ $\uparrow q + 4\gamma = 0 \Rightarrow q = -4\gamma$ (5)

$\rightarrow p + 3\gamma = 0 \Rightarrow p = -3\gamma$ (5)

$2(-4\gamma) - 3(-3\gamma) = 10$

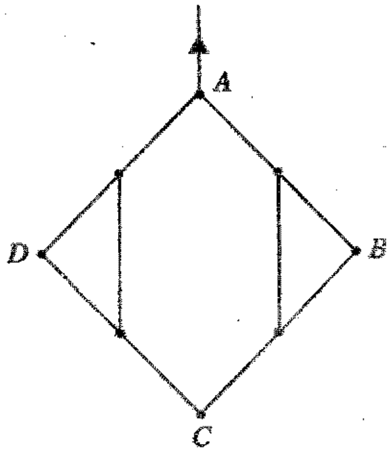
$-8\gamma + 9\gamma = 10$

$\gamma = 10$

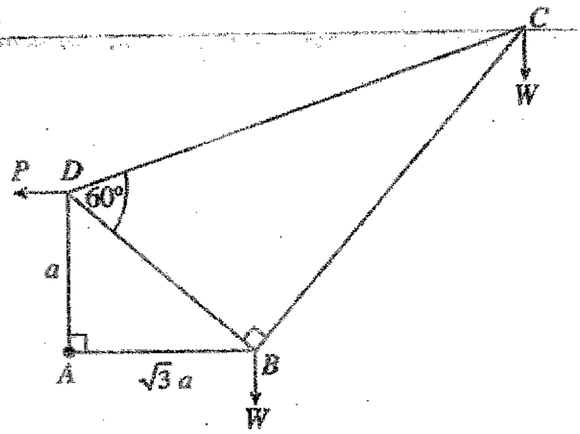
$p = -30$ (5) & $q = -40$ (5)

30

15.(a) Four uniform rods AB, BC, CD and DA each of length $2a$ and weight W are smoothly jointed at their ends A, B, C and D. The midpoints of AB and BC are joined by a light inextensible string of length a. Similarly, midpoints of AD and DC are also joined by a light inextensible string of length a. The system is suspended in a vertical plane from the point A and stays in equilibrium as shown in the figure. Find the tensions in the strings and the reaction exerted on AB by BC at the joint B.



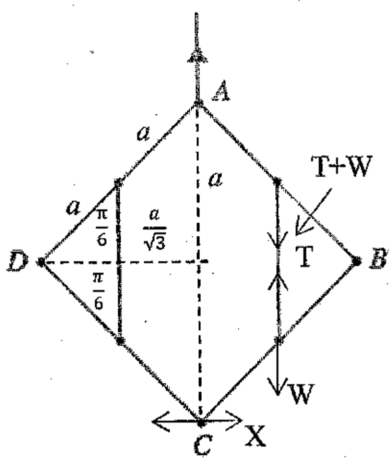
(b) The framework shown in the figure consists of five light rods AB, BC, CD, DA and DB smoothly jointed at their ends. It is given that $AD = a$, $AB = \sqrt{3}\,a$, $B\hat{A}D = 90°$, $C\hat{B}D = 90°$ and $B\hat{D}C = 60°$. At each of the joints B and C, a load W is suspended and the framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with AB horizontal by a horizontal force P applied to it at the joint D.



(i) Find the value of P.

(ii) Draw the stress diagram using Bow's notation for the joints C, B and D.
Hence, find the stresses in the rods, stating whether they are tensions or thrusts.

(a)



சமச்சீரின் படி, DC இனால் CB இற்கு C இல் வழங்கப்படும் மறுதாக்கம் ஆனது கிடையானதாகும்.

(5)

ABC இற்கு,

$$A\curvearrowright : X\cdot 2a-2W\cdot\frac{a\sqrt{3}}{2}=0 \quad (5)$$

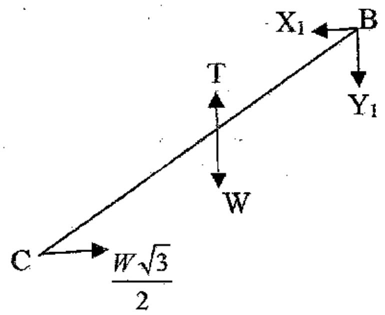
$$X=\frac{\sqrt{3}W}{2} \quad (5)$$

BC இற்கு:

$$B\curvearrowright : \frac{W\sqrt{3}}{2}\cdot a+W\cdot\frac{a\sqrt{3}}{2}-T\cdot\frac{a\sqrt{3}}{2}=0 \quad (10)$$

$$T=2W. \quad (5)$$

BC இற்கு:



$$\rightarrow \quad X_1=\frac{W\sqrt{3}}{2}\ ; \quad (5)$$

$$\uparrow T-W-Y_1=0. \quad (5)$$

$$\therefore Y_1=W \quad (5)$$

$$R=\sqrt{\left(\frac{\sqrt{3}W}{2}\right)^2+W^2}$$

$$=\frac{\sqrt{7}w}{2} \quad (5)$$

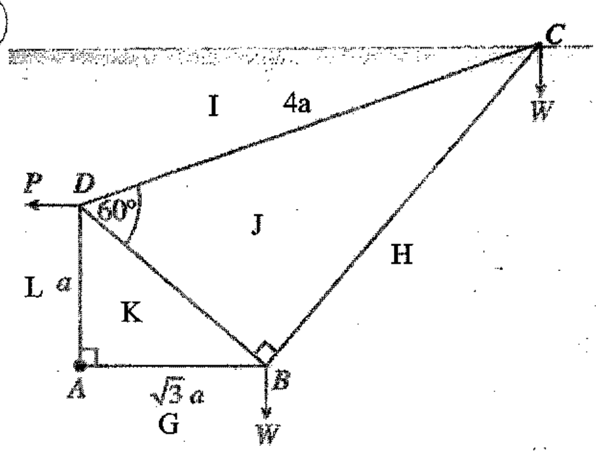
$$\tan\theta=\frac{Y_1}{X_1}-\frac{W}{\frac{W\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}$$

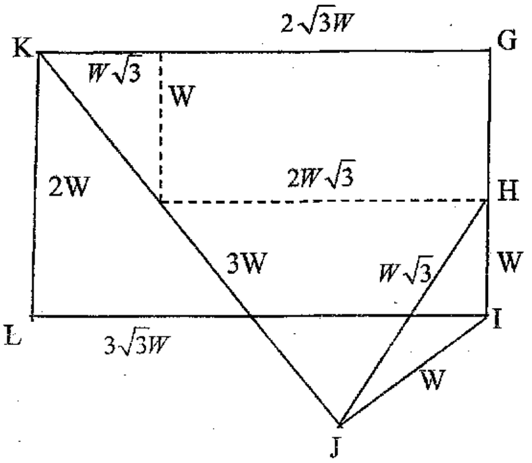
$$\therefore\ \theta=\tan^{-1}\left(\frac{2}{\sqrt{3}}\right) \quad (5)$$

55

(b) $A\circlearrowright$ $P\times a - W\times\sqrt{3}a - W\times 2\sqrt{3}a = 0$ (10)

$\therefore\ P = 3\sqrt{3}W.$ (5)





மூட்டு C: (10)

மூட்டு D: (10)

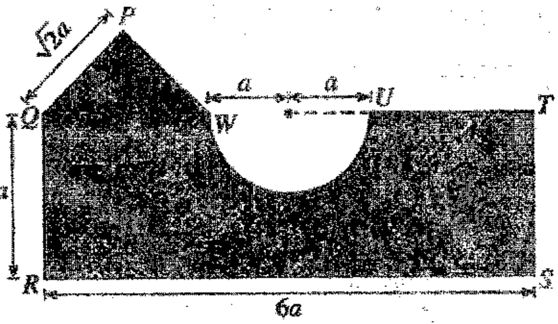
மூட்டு B: (10)

கோல்	தகைப்பு	இழுவை	பருமன்	
AB	✓	-	$3\sqrt{3}W$	(10)
BC	✓	-	$\sqrt{3}W$	(10)
CD	-	✓	W	(10)
BD	-	✓	5W	(10)
AD	✓	-	2W	(10)

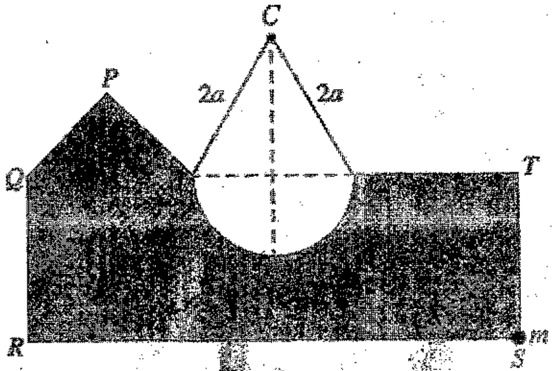
95

16. Show that the centre of mass of a uniform semi-circular lamina of radius r and centre O is at a distance $\frac{4r}{3\pi}$ from O.

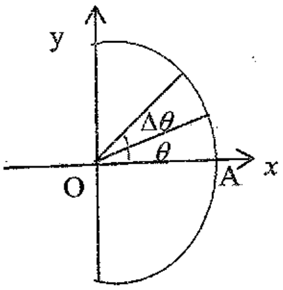
A plane lamina is made from a uniform thin sheet metal of surface density σ by removing a semi-circle of radius a from the rectangle $QRST$ and by adding an isosceles triangle PQW with equal side-lengths $\sqrt{2}a$ to it, as shown in the adjoining figure. $QR = 2a$, $RS = 6a$ and $QW = 2a$. The centre of mass of this lamina lies at a distance $\bar{x}$ from QR and $\bar{y}$ from RS. Show that $\bar{x} = \frac{(74-3\pi)a}{(26-\pi)}$ and $\bar{y} = \frac{2(15-\pi)a}{(26-\pi)}$.



The lamina with a particle of mass m fixed to it at S, hangs in equilibrium by a light inextensible string of length $4a$ whose ends are attached to U and W and passing over a small smooth fixed peg C with side RS horizontal as shown in the figure. Find the value of m and the tension of the string in terms of a and σ.



சமச்சீரின் படி, $\bar{y} = 0$ (5)

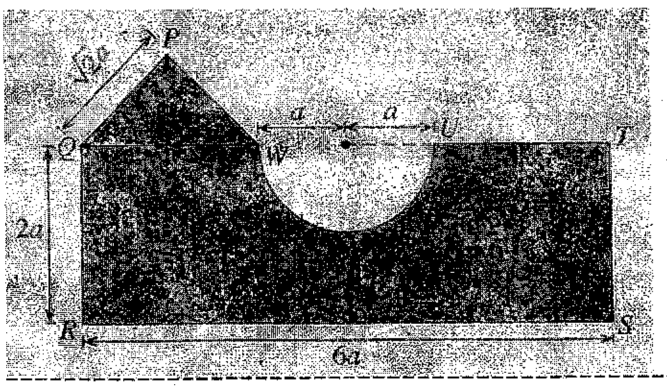


$$\Delta m = \frac{1}{2} r^2 \Delta\theta \times \sigma$$

$$\bar{x} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} r^2 \sigma \cdot \frac{2}{3} r \cos\theta \, d\theta \quad (5)}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} r^2 \sigma \cdot d\theta \quad (5)}$$

$$= \frac{\frac{2}{3} r^3 \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \quad (5)}{\frac{1}{2} r^2 \theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \quad (5)}$$

$$= \frac{4r}{3\pi} \quad (5)$$



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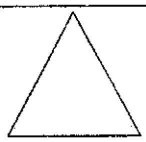
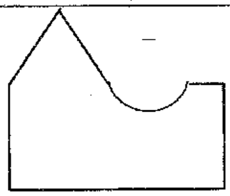
பொருள்	திணிவு	QR இலிருந்து தூரம்	RS இலிருந்து தூரம்	
	$12a^2\sigma$	$3a$	a	(15)
	$\frac{1}{2}\pi a^2\sigma$	$3a$	$2a-\frac{4a}{3\pi}$	(15)
	$\frac{1}{2}(2a)a\sigma$ $=a^2\sigma$	a	$2a+\frac{1}{3}a=\frac{7a}{3}$	(15)
	$12a-\frac{1}{2}\pi i+a^2\sigma$ $\left(13-\frac{\pi}{2}\right)a^2\sigma$ (5)	$\bar{x}$	$\bar{y}$	

$$\left(13-\frac{\pi}{2}\right)a^2\sigma\bar{x}=12a^2\sigma(3a)-\frac{1}{2}\pi a^2\sigma(3a)+a^2\sigma(a) \quad (15)$$

$$\Rightarrow \quad (26-\pi)a^2\sigma\bar{x}=72a^3\sigma3\pi a^3\sigma+2a^3\sigma$$

$$\Rightarrow \quad \bar{x}=\frac{(74-3\pi)a}{(26-\pi)} \quad (5)$$

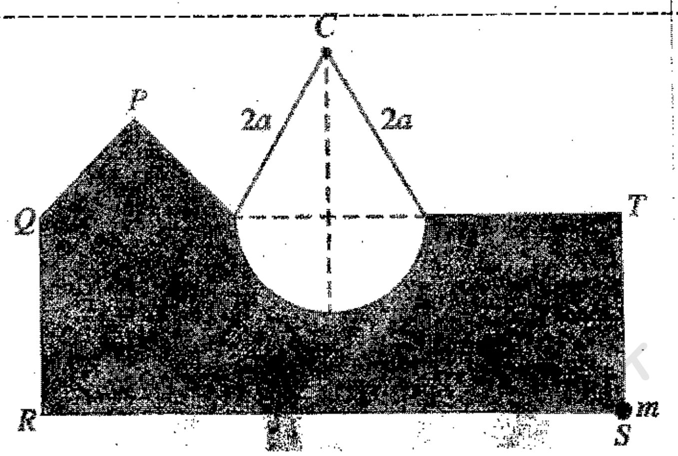
$$\left(13-\frac{\pi}{2}\right)a^2\sigma\bar{y}=12a^2\sigma(a)-\frac{1}{2}\pi a^2\sigma\left(2a-\frac{4a}{3\pi}\right)+a^2\sigma\left(\frac{7a}{3}\right) \quad (15)$$

$$\Rightarrow\left(\frac{26-\pi}{2}\right)a^2\sigma\bar{y}=12a^3\sigma-\pi a^3\sigma+\frac{2a^3\sigma}{3}+\frac{7a^3\sigma}{3} \quad (5)$$

$$=\frac{45a^3\sigma-3\pi a^3\sigma}{3}$$

$$\bar{y}=\frac{2(15-\pi)a}{(26-\pi)} \quad (5)$$

90



C↻ :

$$mg(3a)=\left(13-\frac{\pi}{2}\right)a^2\sigma g(3a-\overline{x})$$ (10)

$$m=\frac{(26-\pi)}{6}a\ \sigma\left(3a-\frac{(74-3\pi)a}{26-\pi}\right)$$ (5)

$$=\frac{a^2\sigma}{2}(4a+3\pi a-3\pi a)$$

$$m=\frac{2a^2\sigma}{3}.$$ (5)

$\uparrow$ $$2T\cos\frac{\pi}{6}=mg+\left(13-\frac{\pi}{2}\right)a^2\sigma g$$ (5)

$\Rightarrow$ $$\sqrt{3}\ T=\frac{2}{3}a^2\sigma g+13a^2\sigma g-\frac{\pi}{2}a^2\sigma g$$

$$=\frac{41a^2\sigma g}{3}-\frac{\pi a^2\sigma g}{2}$$

$$T=\frac{(82-3\pi)a^2\sigma g}{6\sqrt{3}}$$ (5)

30

17.(a) Four identical boxes B_1, B_2, B_3 and B_4, each contains 4 pens which are identical in all aspects except for their colour. Each box B_k contains k red pens and $4-k$ black pens for $k = 1, 2, 3, 4$. One of the four boxes is chosen at random and 2 pens are drawn from that box. Find the probability that

(i) the two pens drawn are red,

(ii) the pens are drawn from box B_4, given that the two pens drawn are red.

(b) The data sets $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_m\}$ have the same mean, and their standard deviations are σ_x and σ_y, respectively. Show that the variance of the combined data set $\{x_1, \ldots, x_n, y_1, \ldots, y_m\}$ is given by $\dfrac{n\sigma_x^2 + m\sigma_y^2}{n+m}$.

Diameters of bolts produced at a factory is summarised in the following table:

Diameter (mm)	Number of bolts (in thousands)
2 – 6	2
6 – 10	5
10 – 14	8
14 – 18	4
18 – 22	1

Estimate the mean, the median and the variance of the distribution given above.

The diameters of another 40 000 bolts produced by a neighboring factory has the same mean, while the variance is 22.53 mm^2. Estimate the combined variance of the diameters of the bolts produced by both factories.

(a)

$$P(RR) = P(RR|B_1)P(B_1) + P(RR|B_2)P(B_2) + P(RR|B_3)P(B_3) + P(RR|B_4)P(B_4) \quad (10)$$

$$= 0\cdot\frac{1}{4} + \frac{^2C_2}{^4C_2}\cdot\frac{1}{4} + \frac{^3C_2}{^4C_2}\cdot\frac{1}{4} + \frac{^4C_2}{^4C_2}\cdot\frac{1}{4} \quad (20)$$

$$= \frac{1}{4\cdot {^4C_2}}[1+3+6]$$

$$= \frac{10}{24} = \frac{5}{12} \quad (5)$$

$$P(B_4|RR) = \frac{P(B_4|RR)P(B_4)}{P(RR)} \quad (10)$$

$$= \frac{1\cdot\frac{1}{4}}{\frac{5}{12}} \quad (5) \quad (5)$$

$$= \frac{12}{20} = \frac{3}{5}. \quad (5)$$

60

(b) தரவுத் தொடைகள் $\{x_1, x_2, \ldots, x_n\}$, $\{y_1, y_2, \ldots, y_m\}$ இன் இடை μ என்க. எனவே இணைந்த தரவுத் தொடையின் இடை μ ஆகும். (5)

$$\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{m} y_i^2}{n+m} - \mu^2 \quad (5)$$

$$= \left[\frac{\sum_{i=1}^{n} x_i^2 - n\mu^2}{n+m}\right] + \left[\frac{\sum_{i=1}^{m} y_i^2 - m\mu^2}{n+m}\right] \quad (5)$$

$$= \frac{1}{n+m}\left[n\left(\underbrace{\frac{\sum_{i=1}^{n} x_i^2}{n} - \mu^2}_{\sigma_x^2}\right) + m\left(\underbrace{\frac{\sum_{i=1}^{m} y_i^2}{m} - \mu^2}_{\sigma_y^2}\right)\right] \quad (5)$$

$$= \frac{n^{\sigma}x^2 + m^{\sigma}y^2}{n+m} \quad (5)$$

25

விட்டம் (mm)	$f(10^3)$	நடுப்பெறுமானம் x	xf	$x^2 f$
2 - 6	2	4	8	32
6 - 10	5	8	40	320
10 -14	8	12	96	1152
14 -18	4	16	64	1024
18 -22	1	20	20	400
	20		228	2928
	(5)		(10)	(10)

இடை $= \frac{\sum xf}{\sum f} = \frac{228}{20} = 11.4\text{mm}$ (5) (5)

Variance $= \frac{\sum x^2 f}{\sum f} - \mu^2 = \frac{2928}{20} - (11.4)^2 = 146.4 - 129.96$ (5) (5)

$= 16.44 \text{ mm}^2$.

Median $= 10 + \frac{(10-7)}{8} \times 4$ (5)

$= 11.5$ mm. (10)

இணைந்த மாறற்றிறன் $= \frac{1}{20+40}\{20\sigma_1^2 + 40\sigma_2^2\} = \frac{1}{60}\{20 \times 16.44 + 40 \times 22.53\}$ (5)

$= 20.5 \text{ mm}^2$.

65