අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය	I
கணிதம்	I
Mathematics	**I**

07 E I

පැය තුනයි	
மூன்று மணித்தியாலம்	
Three hours	

අමතර කියවීම් කාලය	-	මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம்	-	10 நிமிடங்கள்
Additional Reading Time	-	**10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts*;
  Part A (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $A = \{x \in \mathbb{R} : 5 - |2x - 9| \geq 2\}$ and $B = \{x \in \mathbb{R} : |2x - 8| < 6\}$. Find $A \cup B$, $A \cap B$ and $B - A$.

2. Let A, B and C be subsets of a universal set S. Show that $A - (A \cap B \cap C) = A - (B \cap C)$.

[see page three

0388

3. Show that the compound proposition $(p \Rightarrow q) \vee (p \wedge q)$ is logically equivalent to the compound proposition $\sim p \vee q$.

4. Let $n \in \mathbb{Z}$. Using the method of contradiction, prove that if $n^3 + 2n + 7$ is odd, then n is even.

[see page four

5. Obtain a quadratic equation in x from the equation $\log_{10}(x+15) = 2 - \log_{10} x$.
Hence, find the value of x satisfying the equation $\log_{10}(x+15) = 2 - \log_{10} x$.

6. Find all real values of x that satisfy the inequality $\frac{x}{x-2} - \frac{1}{x} < 1$.

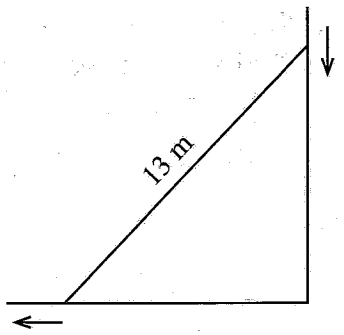
[see page five

7. The functions f and g are defined by $f(x) = 3x + 2$ and $g(x) = \dfrac{6}{2x+3}$.
Find the value of x for which $f(g(x)) = 3$.
Solve the equation $f^{-1}(g(x)) = 1$ for x.

8. Let $A \equiv (5, b)$ and $B \equiv (-1, 3b)$ be two points, where $b \in \mathbb{R}$. Also, let C be the mid-point of AB.
Find the values of b such that AB is perpendicular to OC, where O is the origin.

[see page six

9. A 13 m long ladder is resting on a horizontal ground, while leaning against a vertical wall (see the figure). The foot of the ladder is pulled away from the wall at a rate of 0.2 m s^{-1}. Find the rate at which the top of the ladder is sliding down the wall when the foot of the ladder is at a distance 12 m from the wall.



10. Find the area of the region enclosed by the curves $y = x^2$ and $y = 4x - x^2$.

[see page seven

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024
General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය I
கணிதம் I
Mathematics I

07 E I

Part B

* *Answer **five** questions only.*

11. (*a*) While observing the G.C.E. (Ordinary Level) results for the subjects Mathematics, Science and English of 40 students in a G.C.E. (Advanced Level) class, the following information were revealed:

- 20 students for Mathematics and 18 students for Science have obtained A grades.
- 9 students for both Mathematics and Science, 8 students for both Science and English, and 5 students for both Mathematics and English have obtained A grades.
- 10 students have obtained A grades only for two of the subjects among these subjects.
- The number of students who have not obtained A grades for any of these three subjects is twice the number of students who have obtained A grades for all three subjects.

Find the number of students

(i) who have obtained A grades for all three subjects,

(ii) who have obtained A grade only for Mathematics,

(iii) who have obtained A grade for English.

(*b*) Using truth tables, show that

(i) $(p \wedge q) \vee r$ is logically equivalent to $(p \vee r) \wedge (q \vee r)$,

(ii) $(p \vee q) \wedge (\sim p \vee r) \Rightarrow q \vee r$ is a tautology.

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{(2n+1)} \quad \text{for all } n \in \mathbb{Z}^+.$$

(*b*) Let $f(r) = \dfrac{r}{(r+2)^2}$ for $r \in \mathbb{Z}^+$.

Find U_r such that $U_r = f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$.

Show that $\sum_{r=1}^{n} U_r = \dfrac{1}{9} - \dfrac{(n+1)}{(n+3)^2}$ for $n \in \mathbb{Z}^+$.

Hence, show that $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Also, find $\sum_{r=15}^{\infty} U_r$.

[*See page eight*

13. (*a*) The roots of the equation $x^2 + px + q = 0$ are -3 and 5; where $p, q \in \mathbb{R}$.

Find the values of p and q.

Using these values of p and q, find the value of the constant $r(\in \mathbb{R})$ for which the equation, $x^2 + px + q + r = 0$ has equal roots.

(*b*) Let $p(x) = 2x^3 + ax^2 + (a+4)x + 6$, where $a \in \mathbb{R}$. It is given that $(x-2)$ is a factor of $p(x)$.

Find the value of a.

When a has this value, factorize $p(x)$.

Also, solve the inequality $p(x) > 0$.

14. (*a*) Find the first 3 terms in the expansion of $(1-x-x^2)^6$ in ascending powers of x.

Hence, find the value of k such that the coefficient of x^2 in the expansion of $(k+x^2)(1-x-x^2)^6$ is 10.

(*b*) A person takes a loan of Rs. 500 000 from a bank that charges a monthly interest of 1%. He pays back the loan in equal monthly instalments of Rs. B over 5 years. Let Rs. A_n be the amount to be paid back after n months. Show that

$A_2 = 500\,000\ (1.01)^2 - B(1 + 1.01)$ and

$A_3 = 500\,000\ (1.01)^3 - B(1 + 1.01 + 1.01^2)$.

Write down a similar expression for A_n in terms of $n(\leq 60)$ and B.

Find the value of B.

15. Let the line l_1 be given by $4y = 3x + 1$. The point $A \equiv (a, 4)$ lies on l_1. Find the value of a.

Suppose that the line l_2 is parallel to l_1 and passing through the point $(4, -3)$. Find the equation of l_2.

Let B be the point of intersection of l_2 and the y-axis, and C be the point on l_1 such that BC is perpendicular to l_1. Find the coordinates of C.

Let D be the point such that $ABCD$ is a parallelogram. Find the coordinates of D.

Also, find the area of $ABCD$.

[*See page nine*

16. (*a*) Evaluate $\lim_{x\to 1} \dfrac{(x^3-1)^2}{\left(\sqrt{x}-1\right)\left(2x^2-x-1\right)}$.

(*b*) Differentiate each of the following with respect to x:

(i) $\sqrt{\ln\left(x^4e^x+5x^2+3\right)}$, (ii) $\left(2-3x^2\right)^7\left(x+2x^2\right)^5$, (iii) $\dfrac{3e^{x^2}+2x^3}{3e^{x^2}-2x^3}$.

(*c*) A right circular hollow cylinder, open at one end, is constructed from a thin sheet of metal. The total outer surface area of the cylinder is 192π cm^2. The cylinder has a radius of r cm and a height of h cm.

Express h in terms of r, and show that the volume V cm^3, of the cylinder is given by $V=\frac{1}{2}\pi\left(192r-r^3\right)$.

Find the value of r such that V is maximum.

17. (*a*) Using **partial fractions**, find the value of $\int_1^2 \dfrac{1}{x^2(x+1)}\,dx$.

(*b*) Using **integration by parts**, find the value of $\int_1^2 \left(12x^3+4x\right)\ln x\,dx$.

(*c*) The following table gives the values of the function $f(x) = \ln(4+x^3)$, correct to three decimal places, for values of x between 1 and 2 at intervals of length 0.25:

x	1	1.25	1.5	1.75	2.0
$f(x)$	1.609	1.784	1.998	2.236	2.485

Using **Simpson Rule**, find an approximate value for $\int_1^2 \ln\left(4+x^3\right)dx$.

Hence, find an approximate value for $\int_1^2 \ln\left(\dfrac{1}{4+x^3}\right)dx$.

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General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය	II
கணிதம்	II
Mathematics	**II**

07 E II

පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***	අමතර කියවීම් කාලය - මිනිත්තු 10 යි மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள் **Additional Reading Time - 10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

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* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
* *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II		
Part	**Question No.**	**Marks**
A	**1**	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
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	10	
B	**11**	
	12	
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In Words	

Code Numbers

Marking Examiner		
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	2	
Supervised by:		

[see page two

Part A

1. Let $a \in \mathbb{R}$.

Show that $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1-a^3)^2$.

2. Let $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$. Find $\mathbf{A^2}$.

Find the value of $k(\in \mathbb{R})$ such that $\mathbf{A^2} = k\mathbf{A} - 2\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix.

[see page three

0355

3. The median of the six data 2, 4, x, 7, 10, y arranged in ascending order is 6. It is given that their range is twice the inter-quartile range. Find the values of x and y.
Also, find the mean deviation of the above data.

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4. The mean of 5 observations is 12 and their variance is 2. When another observation is included, the new mean of the 6 observations is 11.
Find the value of this 6^{th} observation included.
Hence, find the new variance.

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[*see page four*

5. Heights of a group of persons are normally distributed with a mean of 58 inches and a standard deviation of 4 inches. Find the probability that the height of a randomly selected person from this group is

(i) at most 60 inches,

(ii) at least 56 inches, given that the person is shorter than 60 inches.

6. The cards in a pack of five cards are labelled as 1, 2, 3, 4 and 5. Cards are randomly drawn from the pack, with replacement, until an even numbered card is obtained. Find the probability that an even numbered card is obtained

(i) at the first draw,

(ii) before the third draw,

(iii) at the third draw.

[see page five

7. The probability that a participant successfully completes Task 1 and Task 2 assigned in a competition are 0.80 and 0.68 respectively. The probability that a participant who is successful in Task 1 will also be successful in Task 2 is 0.7. Find the probability that a participant who is not successful in Task 1 will be successful in Task 2.
Also, find the probability that among three randomly selected participants, no one successfully completes Task 1.

8. Let A and B be two events on a sample space S such that $P(A) = 0.4$, $P\left((A \cup B)'\right) = 0.2$ and $P(A \cap B) = 0.1$.
Find $P(B)$, $P(A \cap B')$ and $P(A \mid B)$.

[see page six

9. The probability mass function of a discrete random variable X is given below:

x	0	1	2	3
$P(X=x)$	a	b	0.3	0.1

The expected value of X is 1.3. Find the values of a and b.
Also, find $P(X \geq 2 \mid X \geq 1)$.

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10. A continuous random variable X has the probability density function given by

$$f(x)=\begin{cases} 2-kx & , \text{ for } 0 \leq x \leq 1 \\ 0 & , \text{ otherwise} \end{cases}$$

Find the values of k and $E(X)$.
Also, find $P(X \leq \frac{1}{4})$.

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General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය II
கணிதம் II
Mathematics II

07 E II

Part B

* *Answer **five** questions only.*

11. A manufacturer produces a certain product in two qualities: Grade I and Grade II. The same raw materials are used to produce both types. The amount of raw materials and labour hours needed to produce one unit in each quality and the profit per unit in each quality are shown in the table below:

	Per unit values	
	Grade I product	Grade II product
Raw materials (kg)	8	10
Labour hours	6	5
Profit (Rs)	140	100

According to the market demand, at least 60 units from each quality must be produced per day.

On each day, the manufacturer has 2 400 kg of raw material and 1 500 labour hours.

It is required to find the number of units to be produced per day from each quality to maximize the total profit.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the optimum solution for this problem.

(iv) During the production of 2 units of Grade II, if 1 unit of a by-product is obtained that gives a profit of Rs. 80 per unit, find the optimum solution under this new situation.

12. (*a*) Let $\mathbf{A}=\begin{pmatrix} 3 & a \\ -4 & 3 \end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix} 4 & b \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$, where $a, b \in \mathbb{R}$.

Find the values of a for which $\mathbf{A}^{-1}$ exists.

Now, let $a = -2$. Write down $\mathbf{A}^{-1}$.

Let $\mathbf{C} = \mathbf{B}\mathbf{A}^{-1}$. Find $\mathbf{C}$ in terms of b.

Find the value of b such that $\mathbf{C}^{\mathrm{T}}\mathbf{B} = \begin{pmatrix} 39 & 15 \\ 25 & 12 \end{pmatrix}$.

(*b*) Let $a, b \in \mathbb{R}$. Write down the system of linear equations

$$ax - y = 2$$
$$4x - 2y = b$$

in the form $\mathbf{AX} = \mathbf{B}$, where $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$, and $\mathbf{A}$ and $\mathbf{B}$ are matrices to be determined.

Show that the above system of equations has

(i) a unique solution, when $a \neq 2$,

(ii) infinitely many solutions, when $a = 2$ and $b = 4$,

(iii) no solutions, when $a = 2$ and $b \neq 4$.

13. (*a*) An unbiased die with faces marked 1, 1, 2, 3, 4, 4 is rolled and an unbiased spinner with six separated regions numbered 1, 2, 3, 2, 2, 1 is spun.

The number on the top face of the die and the number on the region pointed by the arrow of the spinner are recorded.

Let A be the event that both recorded numbers are the same and B be the event that the sum of the two recorded numbers is even.

Find $P(A')$, $P(A' \cap B)$ and $P(A' \cup B)$.

(*b*) Numbers consisting of 5 digits are made by selecting digits from the seven digits from 1 to 7 without repeating any digit.

Find

(i) the total number of different numbers that can be made,

(ii) the number of different numbers that can be made starting with 3, and having 6 and 7 adjacent to each other.

(*c*) A mathematics society consists of 9 senior members and 6 junior members. A research team of 5 people is to be formed to work on a new project.

Find the number of different ways in which the research team can be made

(i) out of all members,

(ii) such that the team has at least 3 senior members.

14. The probability that a school bus arrives on time on a Monday is 0.7. If the bus arrives on time on a particular day, the probability that it will be on time on the following day is 0.8. If the bus is late on a particular day, the probability that it will be late on the following day is 0.4.

Find the probability that the bus arrives on time on a Wednesday.

Also, find the probability that, from Monday to Wednesday of a week, the bus will be

(*a*) late at least on one day,

(*b*) late exactly on two days,

(*c*) late exactly on two days, given that it was late at least on one day.

[*See page nine*

15. The actual weight of cashews in packets labelled as 50 g of a certain cashew distributor follows a normal distribution with a mean of 48 g and a standard deviation of 4 g. A packet of cashews is randomly selected. Let A be the event that the actual weight of cashews in the chosen packet is less than the labelled weight. Let B be the event that the difference between the actual weight of cashews in the chosen packet and the labelled weight is at most 2 g.

Find,

(i) $P(A)$,

(ii) $P(B)$,

(iii) $P(A \cap B')$,

(iv) $P(A|B)$,

and (v) $P(A \cup B)$.

16. The following table summarises the distances travelled by a taxi driver during a certain period of time:

Distance (km)	**No. of days**
10 – 20	9
20 – 30	13
30 – 40	a
40 – 50	16
50 – 60	15
60 – 70	5

(i) If the cumulative frequency corresponding to the 4th class interval is 60, find the value of a.

(ii) Using the transformation $y_i = \frac{x_i - 35}{10}$, or otherwise estimate

(a) total distance travelled, during this period of time,

(b) mean of the distance travelled per day,

(c) median of the distances travelled during this period of time.

(iii) Based on the values of the mean and the median, what can be concluded about the shape of the distribution of the data?

(iv) Sketch the cumulative frequency curve.

(v) Using the curve sketched in part (iv), or otherwise, estimate

(a) the tenth percentile,

(b) the first quartile,

(c) the inter-quartile range

for the given data.

[*See page ten*

17. The duration of activities of a project and the flow of activities are given in the following table:

Activity	Preceding Activity/ Activities	Duration (in months)
A	–	3
B	–	4
C	A, B	5
D	A, C	2
E	C	3
F	B, C	4
G	D, E	5
H	E, F	6
I	G, H	1

(i) Construct the project network.

(ii) Prepare an activity schedule that indicates earliest start time, earliest finish time, latest start time, latest finish time and the float for each activity.

(iii) Find the total duration of the project.

(iv) Write down the critical path of the project.

(v) What are the activities that can be delayed without extending the total duration of the project?

(vi) How is the project completion time affected by delaying the activity F by 2 months?

* * *